Operational Risk Quantification and Insurance

Capital Allocation for Operational Risk

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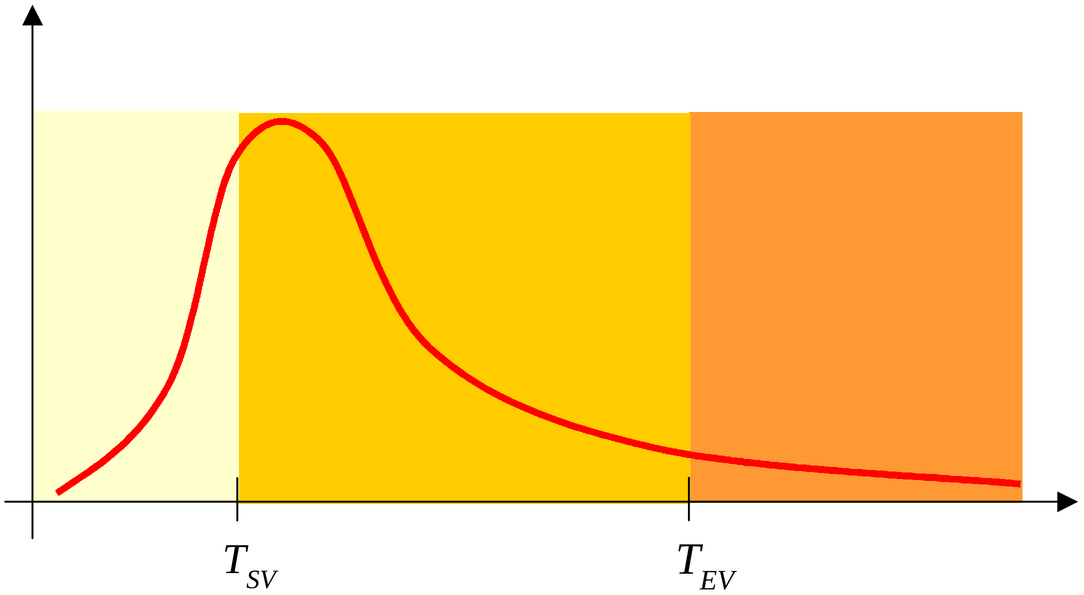
Outline

- Capital Calculation along the Loss Curve
- Hierarchy of Quantitative Models
- Small value modeling
- Extreme value modeling
- An Empirical Study: The Efficacy of Diversification

Capital Calculation along the Loss Curve



T_{SV}	*Small value threshold*
T_{EV}	*Extreme value threshold*

Compute capital for operational risk by introducing a convenience threshold for small value losses and a scenario threshold for extreme value losses.

Hierarchy of Quantitative Models

LDA is based on the distribution estimation of the two random variables of severity and frequency. The estimations rely on loss and exposure data collected by by risk category and by business line.

EVT provides a frame work to estimate the tail of loss distributions.

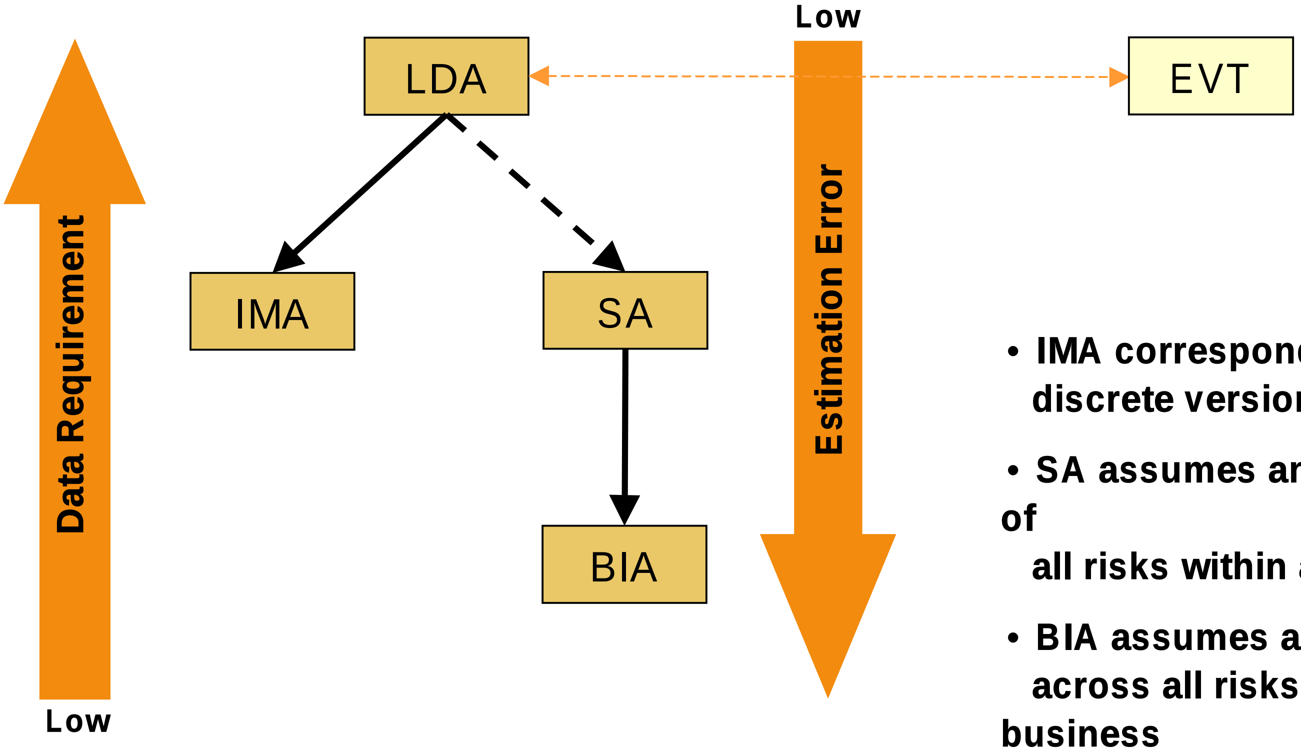
LDA: Loss Distribution Approach

IMA: Internal Measurement approach

SA: Standardized approach

BIA: Basic Indicator Approach

EVT: Extreme Value Theory



- IMA corresponds to a discrete version of LDA
- SA assumes an aggregation of all risks within a business line
- BIA assumes aggregation across all risks and all business lines

Small Value Modeling 1

Aggregate loss describes the total loss incurred within a period of time, usually one year.

The distribution of aggregate loss results from compounding of the distribution of loss severity and that of loss frequency.

Assuming independence of severity and frequency, variance of the aggregate loss can be expressed in terms of mean and variance of severity and frequency.

How to capture the unexpected loss below the threshold T_{SV} ?

Due to high frequency nature, the aggregate distribution below T_{SV} can be approximated by a normal distribution, hence the 99.9 percentile is:

$$UL_{T_{SV}} = 3.1 \times \sigma_S$$

$UL_{T_{SV}}$	Unexpected loss below T_{SV}
σ_S	Standard deviation of aggregate loss

Standard deviation of aggregate loss can be expressed as:

$$\sigma_S = \sqrt{\mathrm{E}[N]\mathrm{VAR}(X) + \mathrm{VAR}(N)\mathrm{E}[X]^2}$$

Small Value Modeling 2

What is the order of magnitude of $UL_{T_{SV}}$?

worst case

Assume all loss be equal to threshold: $X = T_{SV}$

$$UL_{T_{SV}} = 3.1 \times \sigma_N \times T_{SV}$$

Assume a Poisson frequency and

$$\left. \begin{aligned} T_{SV} &= 10'000 \\ E[N] &= 10'000 \end{aligned} \right\} \Rightarrow UL_{T_{SV}} = 3'100'000$$

Small Value Modeling 3

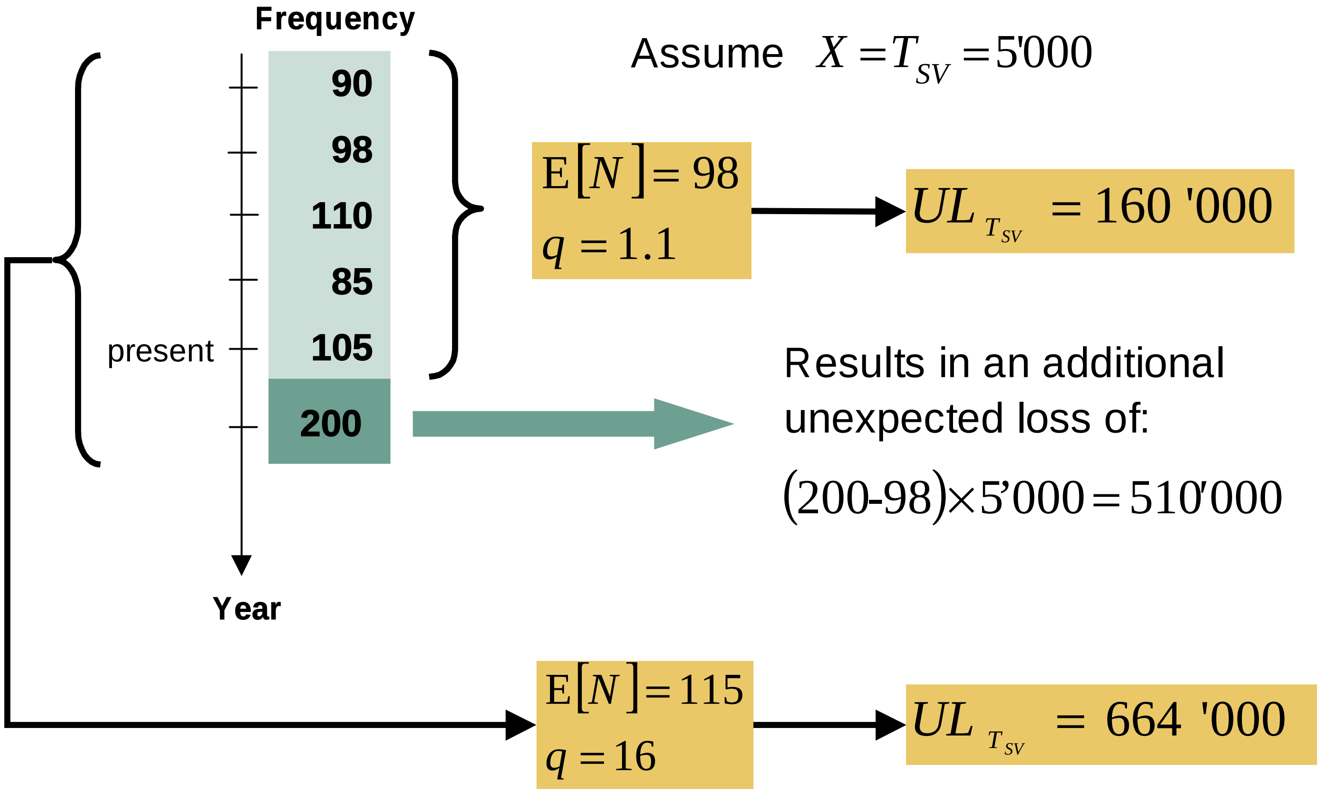
For frequency modeling of small losses it is important to distinguish between a Poisson or a Negative Binomial distribution.

Poisson makes the assumption that losses incur independently. In particular the variance of the frequency is equal to its mean.

In the case of Negative Binomial different events may depend on one and the same cause. At the same time the variance of frequency is assumed to be higher than its mean.

Definition: $q = \frac{\mathrm{Var}(N)}{\mathrm{E}[N]}$

Abrupt changes in the frequency need to be considered!

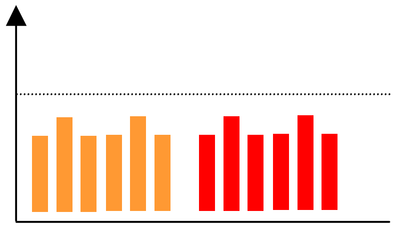


Small Value Modeling 4

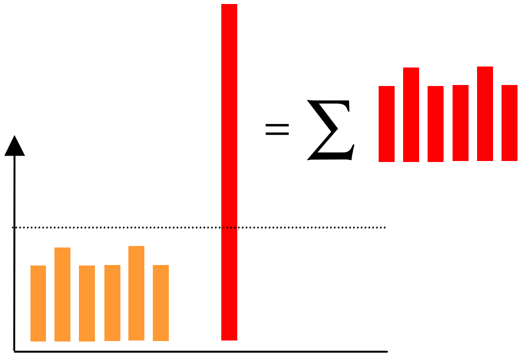
As a rule of thumb the q factor can be determined by assuming an appropriate multiple of the expected frequency (2-3 times) and subsequently computing the implied variance.

How to account for small value events?

- If frequency is stable apply Poisson distribution, otherwise
- assume frequency to be Negative Binomial, or
- if the increase in frequency can be explained by one and the same cause, an aggregation of these events to a single one may be considered



Negative Binomial with $q > 1$



Poisson with $q \approx 1$
Plus a high severity low and frequency scenario

Extreme Value Modeling 1

The general conditions refer to the maximum domain of attraction (MDA) property.

MDA property requires that the distribution of the normalized maxima, for any finite set of samples, converges as the sample size increases.

This property is satisfied for a wide class of distributions applied in insurance.

Notation: $u = T_{EV}$

Pickands-Balkema-de Hann Theorem

Under some general conditions the limiting distribution of the excesses over a high threshold u,

$$F_u(x) = P\left[X - u \le x \middle| X > u\right] = \frac{F(x+u) - F(u)}{1 - F(u)},$$

is either the *Second Pareto* or the *exponential* distribution:

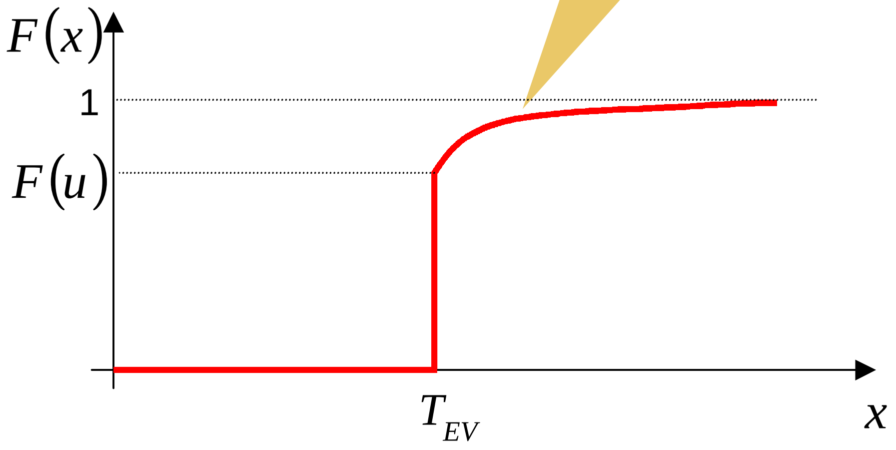
$$F_u(x) \xrightarrow[u \to \infty]{} GP(x) = \begin{cases} SP(x) = 1 - \left(\dfrac{\omega}{\omega + x}\right)^{\rho} \\ E(x) = 1 - \exp(-\rho\ x) \end{cases}$$

Extreme Value Modeling 2

Tail Distribution

The tail of the original distribution above threshold is obtained by fitting either a Second Pareto or an Exponential distribution to the excesses and applying:

$$F(x) = F(u) + \left(1 - F(u)\right) GP(x - u) \quad \text{for} \quad x > u$$



Extreme Value Modeling 3

How to obtain the tail distribution?

Distinguish two cases:

- internal data provides sufficient number of excess losses
- external data is required to gap lack of excess losses

In practice:

- excess losses of different business lines and risk categories may be combined to obtain a sufficient number of excess events

→ one tail model for the organization

Extreme Value Modeling 4

N and F refer to the frequency and severity of internal losses, respectively.

By N_i and E_i we denote the excess frequency and exposure of bank i, respectively.

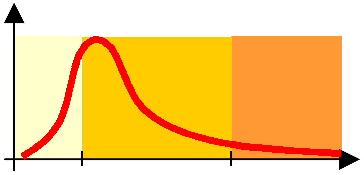
The formula corresponds to the maximum likelihood estimation of a linear exposure model

$$N_{T_{EV}} = f(E) = \alpha E$$

while assuming a Poisson distribution.

How to obtain the tail frequency?

- In case of sufficient internal data, apply the historical excess frequency
- In case of external loss data, either
  - take the extrapolated excess frequency of internal models



Internal LDA model

$$N_{T_{EV}} = N \times (1 - F(T_{EV}))$$

  - or scale the excess frequency of external data by utilizing exposure information

$$N_{T_{EV}} = \frac{\sum_{\text{other banks}} N_i}{\sum_{\text{other banks}} E_i} \times E_{Bank}$$

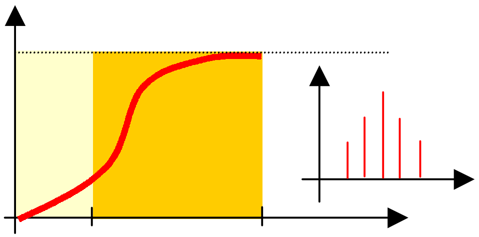
Extreme Value Modeling 5

Aggregation of the internal LDA models and the extreme value model results in the overall loss model.

The overall model can be used to compute relevant quantities such as unexpected loss level.

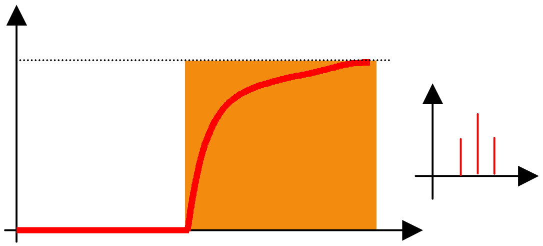
Note that as an alternative we can first compound each severity with the respective frequency distribution and subsequently perform an aggregation of the resulting distributions. However, the latter approach is, however, computationally more costly.

How to bring things together?



Truncated severity distribution and the frequency distribution resulting from internal LDA: (N, F)

+



Excess severity of EVT and excess frequency: $(N_{T_{EV}}, F_{T_{EV}})$

Superposition

$$\begin{cases} N_{total} = N + N_{T_{EV}}, \\ F_{total} = \dfrac{N}{N_{total}} F + \dfrac{N_{T_{EV}}}{N_{total}} F_{T_{EV}} \end{cases}$$

An Empirical Study: The Efficacy of Diversification 1

Data sources

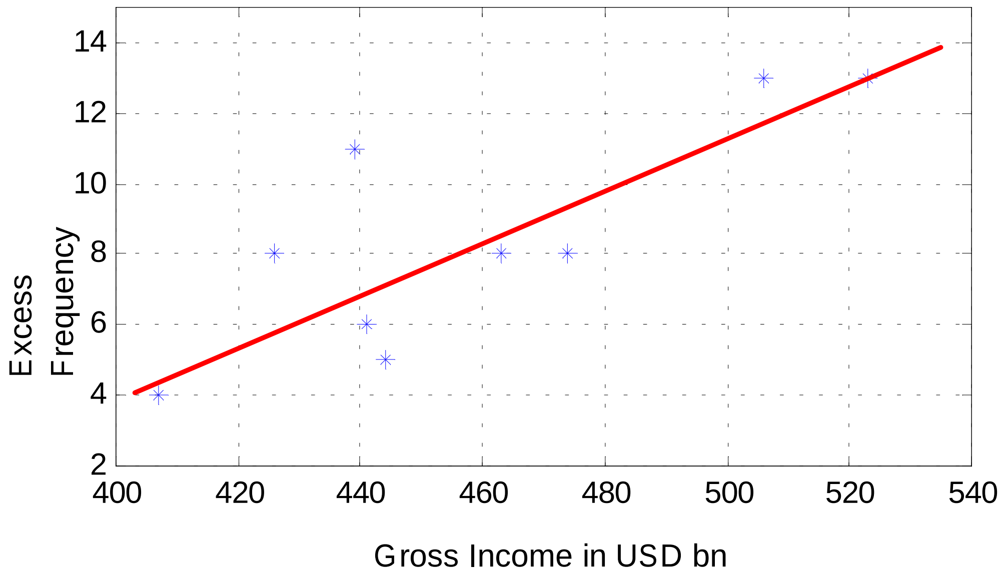
- 84 publicly reported losses in excess of USD 50mn
- OECD Statistics: Bank Profitability 2000
  - Aggregate gross income
  - Aggregate Tier1 and Tier2
- Global Researcher Worldscope data base
  - Balance sheet positions of individual institutions

An Empirical Study: Efficacy of Diversification 2

The linear regression analysis is performed for the aggregate quantities. It suggests that the dependency may also be valid if single institutions are considered.

Similar analysis, however, needs to be conducted to validate the assumption of correlation at the level of single institutions.

Following plot depicts the frequency of losses in excess of USD 50 millions versus gross income of G7 commercial banks



$$Freq = 0.07 \times GI - 23.5$$

$$R^2 = 64\%$$

An Empirical Study: Efficacy of Diversification 3

The frequency of losses in excess of USD 50mn for year 1998 is estimated by applying the linear regression analysis to the gross income of 1998. This results in 11.7 losses.

An estimation of the number of losses below the threshold of USD 50 millions is required to estimate the tail of the original distribution. Here we made the two assumptions of 99.5% and 99%. E.g., 99.5% implies that the 11.7 estimated losses constitute only 0.5% of observable events.

By applying EVT distributions we obtain:

$$SP(x) = 1 - \left(\frac{194377}{194377 + x} \right)^{1.507}$$

As an example we compute the capital at 99 percentile for year 1998 assuming the following two scenarios:

$$N_{T_{EV}} = 11.7$$

$$F(T_{EV}) = 99.5\,\% \Rightarrow UL_{div} = 12'110 \text{ mn}$$

$$F(T_{EV}) = 99\,\% \Rightarrow UL_{div} = 17'040 \text{ mn}$$

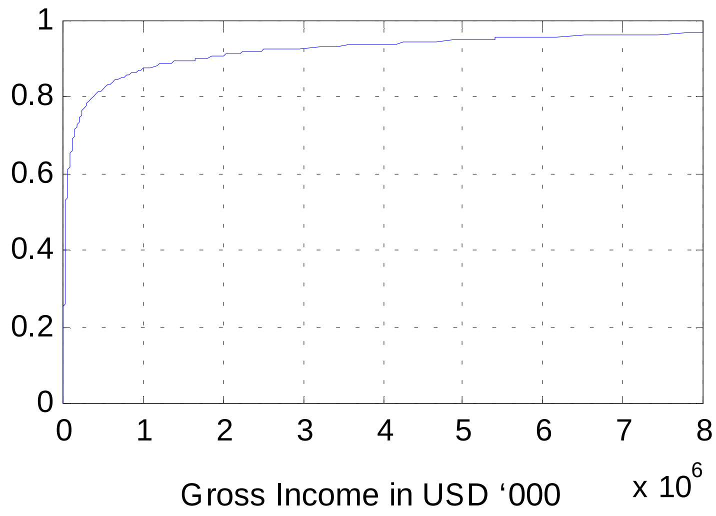
These figures already take into account the effect of diversification!

An Empirical Study: Efficacy of Diversification 4

The distribution of gross income is derived by utilizing the world scope data base. Gross income data of 842 commercial banks world wide were utilized to derive the empirical distribution depicted.

To obtain the undiversified capital, we need to compute the gross-income-weighted number of institutions.

The number of commercial banks in G7 for year 1998 was **9’862**.



From the distribution of gross income we obtain approximately **450** institutions as the gross-income-weighted number.

An Empirical Study: Efficacy of Diversification 5

To undo the effect of diversification, we assume independence of organizations and apply the the square root property known for standard deviation of independent and identically distributed random variables.

The undiversified capital is computed by means of

$$UL = \sqrt{450} \times UL_{div}$$

For comparison consider the ratio of capital to current Tier1+Tier2 level

	Diversified	Undiversified
$F(T_{EV}) = 99\%$	1.3%	27%
$F(T_{EV}) = 99.5\%$	0.9%	20%

Previous analysis suggests a risk sensitive calibration of BIA.

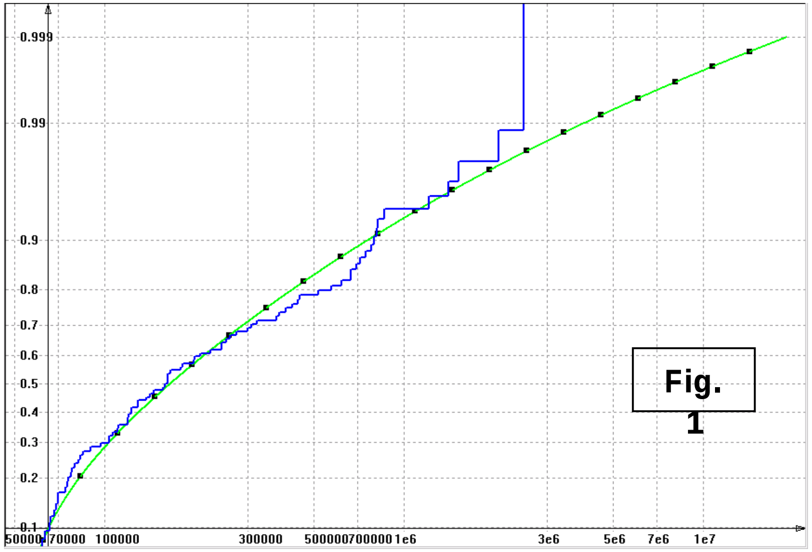


Fig. 1

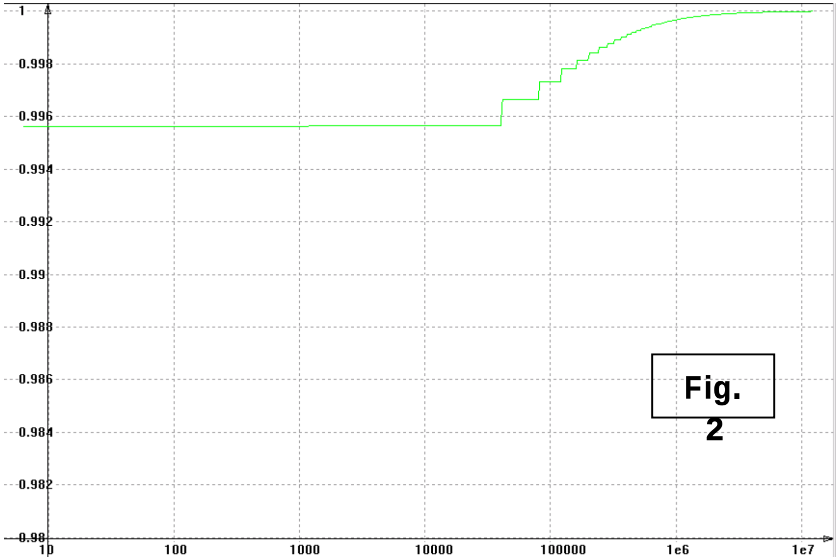


Fig. 2

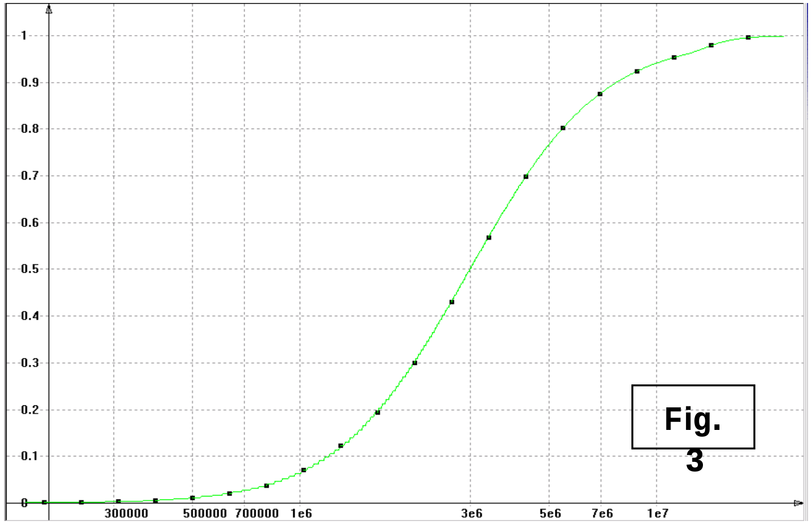


Fig. 3

Fig.1: Second Pareto fit of excess losses

Fig.2: Original distribution recovered by assuming that 99.5% of losses are below the excess threshold

Fig.3: Aggregation of distribution in Fig.2 with the estimated frequency 11.7/(1-0.995)

An Empirical Study: Efficacy of Diversification 6

An analogous example, is provided by insurance linked securitization.

In contrast to risk transfer to an insurer, securitized risks are transferred to capital markets. In case of a securitization the amount of funds provided by investors are equal to capacity guaranteed by the transaction.

Whereas, the same risk, if transferred to an insurer, would require less "funds."

Insurance as an intermediate solution between the diversified and undiversified worlds achieves the diversification benefit in an efficient way