On Transition Probabilities of Regime Switching in Electricity Prices *

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First draft: August 24, 2004
This draft: December 20, 2004

ABSTRACT

In this paper, we analyze transition probabilities of regime switching in electricity prices based on supply and demand using the structural model of Kanamura and Ōhashi (2004). We show that the transition probabilities depend on the demand level and thus are not constant. This result contrasts sharply with the results of many electricity price models that assume constant transition probabilities among different regimes. We also estimate the model using historical data from the PJM market, and empirically analyze the seasonality of the transition probabilities. The results obtained here are consistent with the observed characteristics of price spikes in electricity markets where spikes tend to occur in summer and winter when the demand level is high. These results support the argument by Lucia and Schwartz (2002) that asserts the importance of seasonality in modeling electricity prices.

Key words: Electricity, Price Spikes, Transition Probabilities
JEL Classification: C61, D21, G13, L94, Q40

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*We are grateful to Toshiki Honda, Ronald Huisman, Ryuzo Miura, Izumi Nagayama, and Nobuhiro Nakamura for their helpful comments. Views expressed in this paper are those of the authors and do not necessarily reflect those of JPOWER. All remaining errors are ours.

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1. Introduction

Four unique characteristics are often used to describe electricity prices : mean reversion, seasonality, stochastic volatility and spikes. Of these, spikes are the most important characteristic for risk management in electricity markets. Thus many models have been developed to describe them.

These models formulate price spikes using jump diffusion processes and/or regime switching between a non-spike regime (where prices are unlikely to spike) and a spike regime (where prices are likely to spike). For example, Johnson and Barz (1999) analyze four types of electricity price jump diffusion models. Deng (2000) proposes more sophisticated mean-reverting jump diffusion models with deterministic/stochastic volatility and regime switching. Huisman and Mahieu (2001) develop a regime-switching model with a mean-reversion regime and two different jump diffusion regimes, and Jong and Huisman (2002) develop a regime-switching model with a mean-reverting regime and a jump regime without using a jump diffusion process. Furthermore, Thompson, Davison, and Rasmussen (2003) formulate electricity prices using a jump diffusion process with regime switching.

For tractability, most regime-switching models assume constant transition probabilities from one regime to another, e.g., from a non-spike regime to a spike regime. [1] This simplification allows the models to incorporate the magnitude and frequency of price spikes easily. However, it may not necessarily fit the observed facts. In reality, price spikes usually occur in summer and sometimes in winter when electricity demand is so high that it exceeds the normal base-load supply capacity due to hot (or cold) weather. The spikes also occur due to supply shortages caused by the unavailability of certain power generators. Thus, if the demand level is low compared with the normal supply capacity, prices are unlikely to spike, but as demand increases and approaches supply capacity, prices become more likely to spike. In this sense, the transition probability from a non-spike regime to a spike regime cannot be constant, but depends on the demand level compared to supply capacity.

In this paper, we explicitly incorporates the relation between demand/supply and price spikes in electricity markets by using the structural model of Kanamura and Ōhashi (2004), and obtain the transition probabilities between a non-spike regime and a spike regime. The result shows that the transition probabilities are not constant, but depend on the current demand level, the deterministic trend in demand change, and the trend caused by the deviation of temporary demand fluctuation from its long-term mean. This indicates the importance of the demand level as a state variable in describing the transition probabilities, and suggests the direction of further development in current regime-switching models on electricity prices. We also empirically estimate the model, calculate the transition probabilities, and show how accurately the model can capture the transition probabilities of regime switching compared with other models with regard to time dependency and seasonality.

This paper is organized as follows: Section 2 formulates the model of transition probabilities of regime switching based on the structural model of electricity prices by Kanamura and Ōhashi (2004). Section 3 empirically estimates the model by using historical data from the PJM market and calculates the regime-switching transition probabilities. Section 4 concludes and offers further discussions.

2. The Model

The structural model proposed by Kanamura and Ōhashi (2004) is based on a simple economic idea. Demand and supply determine price. Demand for electricity fluctuates in the short run as underlying factors, such as temperature, fluctuate, while supply is relatively stable. Since the supply curve is upward sloping, price rises when demand increases. However, when the supply reaches a threshold, say the capacity of normal power generation, the marginal cost of electricity generation, i.e., the slope of the supply curve, suddenly and drastically increases. Consequently, when demand exceeds that threshold, the equilibrium price of electricity suddenly and drastically increases. This causes a spike (or “jump”) in electricity prices. The transition probability from a non-spike regime to a spike regime in the structural model is thus obtained by the probability of demand reaching the threshold of supply.

Below, we first formulate the demand and supply curves for electricity, and obtain the equilibrium price. We then characterize the price spikes and the transition probabilities from a non-spike regime to a spike regime.

2.1. The Demand Curve

Electricity demand has two characteristics. First, its fluctuation is strongly affected by temperature, which follows a mean-reverting process around its normal level. Second, in the short run, it is inelastic to price. As a simple model of demand that satisfies these characteristics, Kanamura and Ōhashi (2004) formulate electricity demand as follows:

$$D_t = \bar{D}_t + X_t \tag{1}$$

$$dX_t = (\mu_X - \lambda_X X_t)dt + \sigma_X dw_t, \tag{2}$$

where t denotes the date from the beginning of a given year, D_t denotes the electricity demand on date t, $\bar{D}_t$ denotes the normal level of demand on date t, and X_t denotes the temporary deviation of D_t from $\bar{D}_t$. Note that the normal level of demand $\bar{D}_t$ describes the seasonality of demand for electricity and can be calculated using the average level of past demand on date t.

2.2. The Supply Curve

We assume that power companies supply electricity competitively. [2] Hence, the supply curve corresponds to the upward sloping marginal cost curve of power generation.

Figure 1 shows scatter plots between demand (i.e., supply) levels and prices in the PJM electricity market from January 1, 1999 to December 31, 2000. These scatter plots appear to

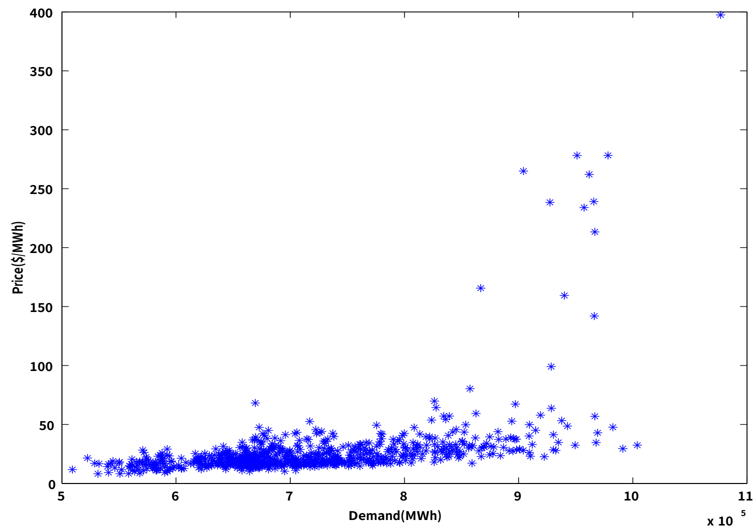

Figure 1. Hockey Stick Model

be mapped on two different lines with different slopes. Below 900,000MWh, the line is flat and the increase in demand (i.e., supply) leads to a small rise in prices. Above that threshold, however, the line becomes much steeper and increasing demand (i.e., supply) leads to a sudden and huge price rise.

Such a sudden rise in price occurs due to the increase in the marginal cost of supply. When the demand level is relatively low, power companies can use their base-load facilities, such as nuclear and coal-fired plants that have low marginal costs, to generate electricity. When demand increases and exceeds the capacity of the base-load facilities, the companies have to start operating their peak-load facilities, such as oil-fired plants that have much higher and accelerating marginal costs. This causes a kink in the supply curve and hence spikes in electricity prices.

Denote by P_t the electricity price and by S_t the supply. The structural model formulates the supply curve with a hockey stick shaped line that consists of two lines with different slopes connected by a quadratic curve, as follows:

$$P_t = f(S_t) = \alpha_1 + \beta_1 S_t + \varepsilon_t \qquad (S_t \leq z - s) \tag{3}$$

$$P_t = f(S_t) = a + bS_t + cS_t^2 + \varepsilon_t \qquad (z - s < S_t < z + s) \tag{4}$$

$$P_t = f(S_t) = \alpha_2 + \beta_2 S_t + \varepsilon_t \qquad (S_t \geq z + s) \tag{5}$$

where $\quad x_1 = z - s, \quad x_2 = z + s, \quad a = \alpha_1 + \beta_1 x_1 - bx_1 - cx_1^2$

$$b = \frac{x_2\beta_1 - x_1\beta_2}{x_2 - x_1}, \quad c = \frac{\beta_2 - b}{2x_2}, \quad \alpha_2 = -\beta_2 x_2 + a + bx_2 + cx_2^2.$$

The two lines and the quadratic curve are connected at two points $z - s$ and $z + s$ where z denotes the middle point of the domain of the quadratic curve.

2.3. Equilibrium Price

Equilibrium electricity prices are obtained by setting supply equal to demand. That is, we set $S_t = D_t$ on the supply curve and obtain equilibrium prices as $P_t = f(S_t) = f(D_t)$. From equation (3) to (5), we have

$$P_t = f(D_t) \tag{6}$$

$$D_t = X_t + \bar{D}_t \tag{7}$$

$$dX_t = (\mu_X - \lambda_X X_t)dt + \sigma_X dw_t \tag{8}$$

where f denotes the supply function of a hockey stick curve and D_t denotes demand for electricity.

2.4. Price Regimes and Transition Probabilities

Recall that in the definition of the supply curve, $z-s$ is the supply level at which the increase in the marginal cost of power generation starts accelerating. Let $P(z-s)$ be the corresponding price. We define the non-spike regime of electricity prices using the set of prices less than or equal to $P(z-s)$. Also, we define the spike regime of electricity prices using the set of prices greater than $P(z-s)$. [3] [4]

More precisely, we say that prices are in the non-spike regime when they are in the range of equation (9) and in the spike regime when they are in the range of equation (10) or equation (11).

$$\text{Non-spike Regime:} \quad P_t = f(D_t) = \alpha_1 + \beta_1 D_t + \varepsilon_t \qquad (D_t \leq z-s) \tag{9}$$

$$\text{Spike Regime:} \quad P_t = f(D_t) = a + bD_t + cD_t^2 + \varepsilon_t \qquad (z-s < D_t < z+s) \tag{10}$$

$$P_t = f(D_t) = \alpha_2 + \beta_2 D_t + \varepsilon_t \qquad (D_t \geq z+s) \tag{11}$$

Roughly, in the non-spike regime, electricity is generated using base-load facilities with low marginal costs, while in the spike regime, it is generated using peak-load facilities with high and accelerating marginal costs.

We are interested in the transition probabilities of regime switching in electricity prices that we define by the probabilities of price at time $t+1$ in one regime conditional on the price at time t in the other regime. Since electricity prices are transformed into demand by a non-decreasing function, the probabilities can be expressed as the function of demand.

Proposition 1

Define

$$\mu_D = D_t + (\bar{D}_{t+1} - \bar{D}_t) + (\mu_X - \lambda_X X_t)\frac{1-e^{-\lambda_X}}{\lambda_X} \tag{12}$$

$$\sigma_D = \sigma_X \sqrt{\frac{1-e^{-2\lambda_X}}{2\lambda_X}}. \tag{13}$$

Then, the transition probability from the non-spike regime to the spike regime (π_{NS}) from t to $t+1$ is given by

$$\pi_{NS} = \int_{z-s}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_D} e^{-\frac{(y-\mu_D)^2}{2\sigma_D^2}} dy \qquad (D_t < z - s). \tag{14}$$

Also, the transition probability from the spike regime to the non-spike regime (π_{SN}) from t to $t+1$ is given by

$$\pi_{SN} = \int_{-\infty}^{z-s} \frac{1}{\sqrt{2\pi}\sigma_D} e^{-\frac{(y-\mu_D)^2}{2\sigma_D^2}} dy \qquad (D_t \geq z - s). \tag{15}$$

The proof is shown in Appendix A.

Note that the transition probabilities from one regime to the other are not constant, but depend on the current demand level D_t, the deterministic trend of demand change $\bar{D}_{t+1} - \bar{D}_t$, and the trend caused by the deviation of the temporary demand fluctuation from its long-term mean $(\mu_X - \lambda_X X_t)\frac{1-e^{-\lambda_X}}{\lambda_X}$. This is natural given the facts that price spikes tend to occur in the hot summer and the cold winter when the demand level is unusually high, and that whether the demand reaches the threshold value of supply in a particular period depends on the demand trend.

This result shows the importance of the demand level as a state variable in describing the transition probabilities, and suggests the direction of further developments in current regime-switching models of electricity prices, many of which assume constant transition probabilities.

3. Empirical Studies

We now empirically estimate the structural model, calculate the transition probabilities, and show how accurately the structural model can capture the transition probabilities of regime switching compared with other models with regard to time dependency and seasonality.

3.1. Data

We use daily average prices and daily demand data calculated using the hourly prices and demand in the PJM electricity market. We do not employ hourly data in this paper because our objective is not to investigate the intra-daily characteristics of prices, but to analyze the seasonality of transition probabilities. We use the prices in the PJM Western Hub as a proxy for the entire PJM market. The data length is from April 1, 1998 to March 31, 2002.

The descriptive statistics are presented in Table 1. We find a large standard deviation,

	Electricity Price ($\$/MWh$)	Electricity Volume (MWh)
Average	26.8	714770.0
Standard Deviation	25.0	95596.4
Standard Error	0.65	2501.0
Sample Number	1461	1461
Variance	624.2	9.1×10^9
Skewness	8.0	0.7
Kurtosis	80.8	0.5

Table 1
Descriptive Statistics of Electricity Price and Volume

skewness, and kurtosis of the electricity prices. On the other hand, the distribution of the volume has a smaller skewness and kurtosis and is closer to the normal distribution than that of price.

3.2. Parameter Estimation

3.2.1. Estimation of Demand Process Parameters

In order to estimate the parameters of the demand process, we express the deviation X_t of the electricity demand from its average using an AR(1) process in the interval of $\Delta t = 1$,

$$\Delta X_t = \alpha_0 + \alpha_1 X_t + \nu_t \tag{16}$$

where $\nu_t \sim N(0, \sigma_\nu^2)$ is disturbances. We denote the set of parameters by $\Theta = (\alpha_0, \alpha_1, \sigma_\nu^2)$, and estimate them using the maximum likelihood method with the initial values obtained from the least square method. The estimates are presented in Table 2. Both parameters of α_1 and σ_ν^2

Parameter	α_0	α_1	σ_ν^2
Estimate	-19.40	-0.33	$2.32 * 10^9$
t-statistic	-0.02	-37.72	28.85
Log-likelihood	-17813.83		
SIC	35633.67		
AIC	35649.52		

Table 2
Parameter Estimates for PJM Demand

are statistically significant in t-statistics, while α_0 is not. This result reveals that the mean-reverting property (represented by α_1) is comparatively strong.

We transform these estimates in a discrete time model to those in a continuous time model using the following transformation (e.g., Clewlow and Strickland (2000)). Then, the parameters $(\mu_X, \lambda_X, \sigma_X)$ in equation (8) are given by

$$\lambda_X = -\log(1+\alpha_1), \quad \mu_X = \frac{\alpha_0}{\alpha_1}\log(1+\alpha_1), \quad \sigma_X = \sigma_\nu \sqrt{\frac{2\log(1+\alpha_1)}{(1+\alpha_1)^2 - 1}}$$

where σ_ν is the standard deviation of the errors ν_t in equation (16). From Table 2, we obtain

$$\lambda_X = 0.40, \quad \mu_X = 0, \text{ and } \sigma_X = 58139.83.$$

3.2.2. Estimation of Supply Curve Parameters

As in Figure 1, the supply curve has different slopes below and above 900,000MWh, reflecting the constitution of power plants. We employ the hockey stick model to represent such characteristics of the supply curve. We estimate the parameters of the model using a hockey stick regression, where the data are the PJM daily average price and daily demand from January 1, 1999 to December 31, 2000. Using the short period data for two years so as to keep the constitution of power plants fixed in the market, we apply a nonlinear least square method to estimate the parameters. The result is presented in Table 3. Judging from the t-statistics, all

Parameter	α_1	β_1	β_2	z	s
Estimate	-20.31	$6.21 * 10^{-5}$	$1.53 * 10^{-3}$	$9.03 * 10^{5}$	$3.43 * 10^{4}$
t-statistic	-2.68	5.77	10.06	124.55	1.89
Log-likelihood	-3292.03				
SIC	6617.03				
AIC	6594.06				

Table 3
Estimation of Supply Curve Parameters

parameters except the parameter s are statistically significant.

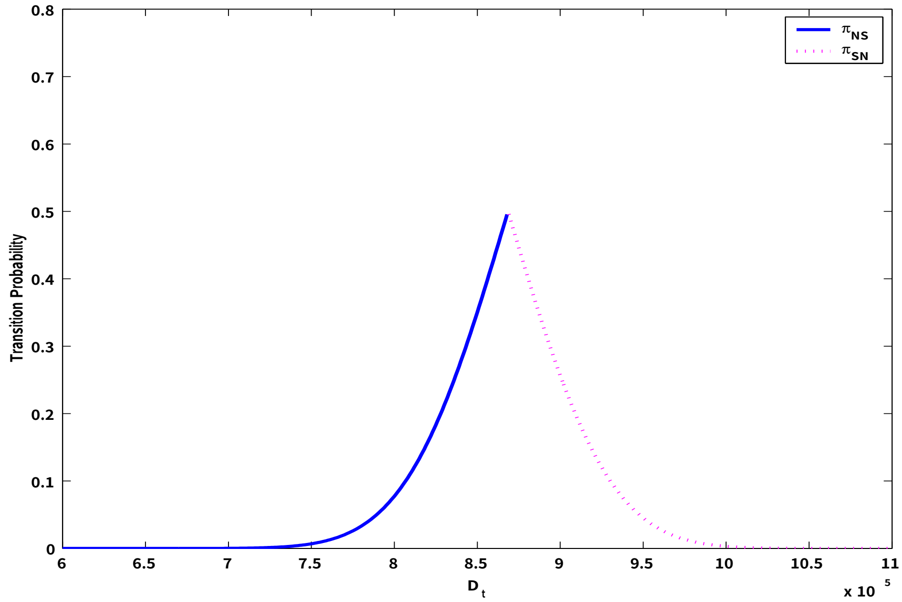

Figure 2. Transition Probabilities ($\bar{D}_{t+1} - \bar{D}_t = 0$, $(\mu_x - \lambda_x X_t)\frac{1-e^{-\lambda_X}}{\lambda_X} = 0$)

The shape of the hockey stick model shows that the degree of price changes is small in the low-demand (non-spike) region, while it is large in the high-demand (spike) region. Indeed, an increase in demand (i.e., supply) from 700,000MWh to 800,000MWh increases price only by \$6$/MWh$, while an increase in demand from 1,000,000MWh to 1,100,000MWh increases the corresponding price dramatically by \$153$/MWh$. This large difference leads to the price spikes.

3.3. Comparative Statics of Transition Probabilities

Figure 2 shows the effect of the current demand level D_t on the transition probabilities. As estimated, $\lambda_X = 0.40$, $\mu_X = 0$, and $\sigma_X = 58139.83$. We set the deterministic trend of demand change $\bar{D}_{t+1} - \bar{D}_t$ and the trend caused by the deviation of the temporary demand fluctuation from its long-term mean $(\mu_X - \lambda_X X_t)\frac{1-e^{-\lambda_X}}{\lambda_X}$ equal to zero, and then plot the transition probabilities for each demand level D_t.

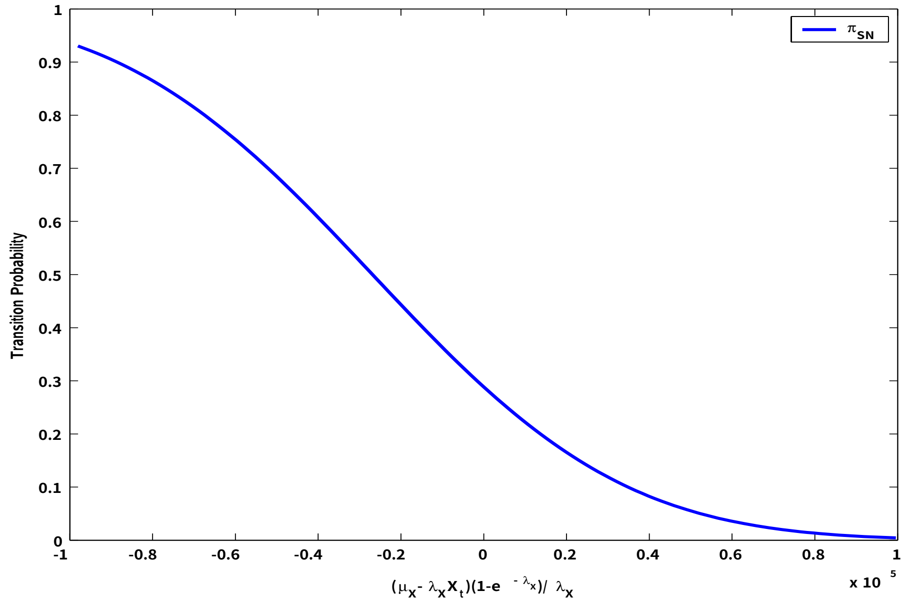

Figure 3. Transition Probabilities ($D_t = 900,000$, $\bar{D}_{t+1} - \bar{D}_t = 0$)

The solid line shows the transition probability π_{NS} from the non-spike regime to the spike regime and the dotted line shows the transition probability π_{SN} from the spike regime to the non-spike regime. In this case, since the demand in one period is normally distributed with zero mean, the maximum value of the transition probabilities is 0.5. As indicated, the demand level D_t affects both of the transition probabilities significantly.

Figures 3 and 4 show the effects of the trend caused by the deviation of the temporary demand fluctuation from its long-term mean $(\mu_X - \lambda_X X_t)\frac{1-e^{-\lambda_X}}{\lambda_X}$ to the transition probabilities π_{SN} and π_{NS}, respectively. To set the current demand in the spike regime, we set D_t equal to 900,000 MWh in Figure 3, whereas in Figure 4, we set D_t equal to 850,000 MWh, which is in the non-spike regime.

The increase in the trend caused by the deviation $(\mu_X - \lambda_X X_t)\frac{1-e^{-\lambda_X}}{\lambda_X}$ decreases the transition probability π_{SN} from the spike regime to the non-spike regime, and increases the transition probability π_{NS} from the non-spike regime to the spike regime. Note that both of the current demand levels 900,000 MWh and 850,000 MWh taken here are close to the threshold

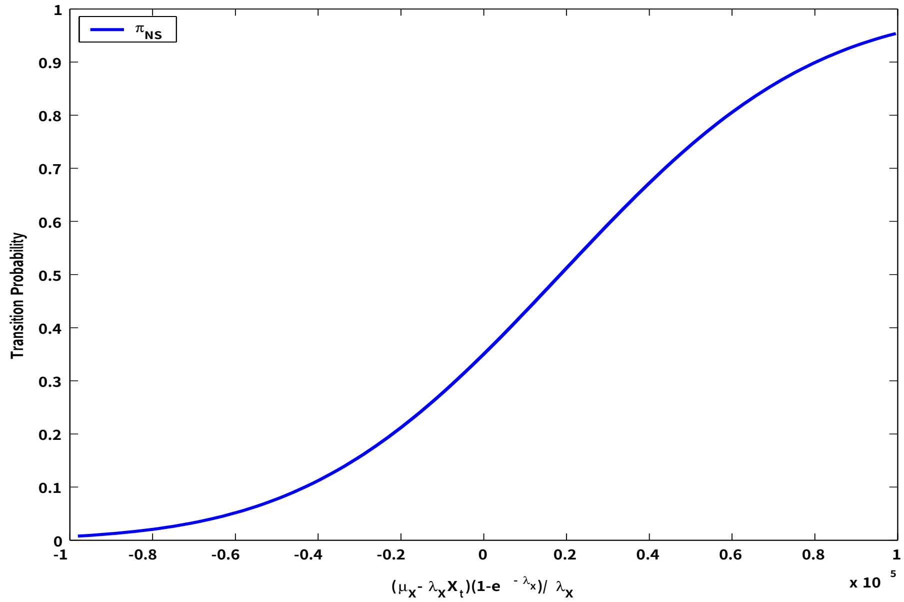

Figure 4. Transition Probabilities ($D_t = 850,000$ and $\bar{D}_{t+1} - \bar{D}_t = 0$)

$z - s = 868,520$ MWh. This is why the effects are significant. One can easily show that the wider the distance between current demand D_t and the threshold $z - s$, the less significant the effect of the trend caused by the deviation on the transition probabilities.

Figures 5 and 6 show the effects of the deterministic trend of demand change $\bar{D}_{t+1} - \bar{D}_t$ on the transition probabilities π_{SN} and π_{NS}, respectively. In Figure 5, we set D_t equal to 900,000 MWh, which is in the spike regime, and in Figure 6, we set D_t equal to 850,000 MWh, which is in the non-spike regime. Again, the increase in the deterministic trend of demand $\bar{D}_{t+1} - \bar{D}_t$ decreases the transition probability π_{SN} from the spike regime to the non-spike regime, and increases the transition probability π_{NS} from the non-spike regime to the spike regime.

3.4. Seasonality of Transition Probabilities

We simulate the transition probabilities to see how they vary through time. The probabilities (π_{NS} and π_{SN}) and the corresponding average demand ($\bar{D}_t$) are illustrated in Figure 7

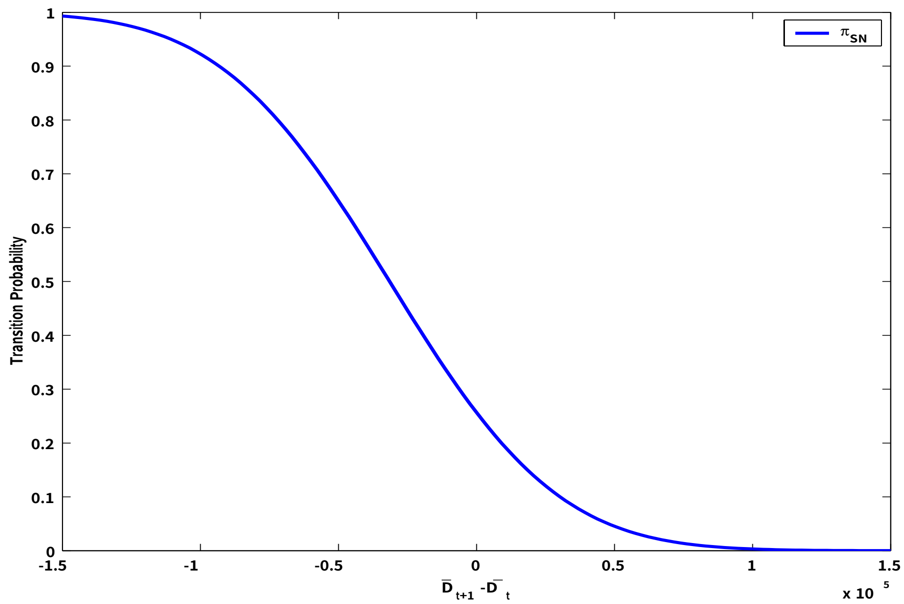

Figure 5. Transition Probabilities ($D_t = 900,000$, $(\mu_x - \lambda_x X_t)\frac{1-e^{-\lambda_X}}{\lambda_X} = 0$)

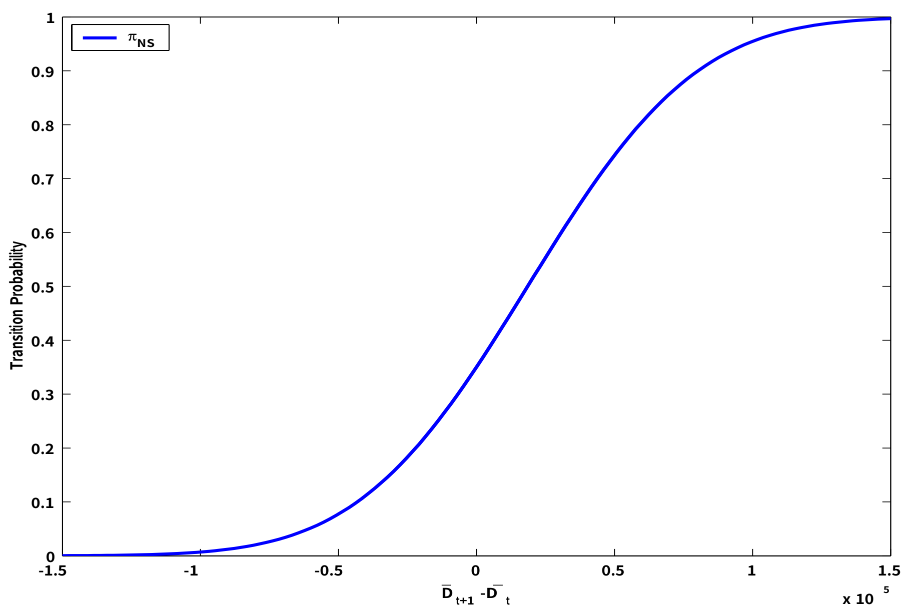

Figure 6. Transition Probabilities ($D_t = 850,000$, $(\mu_x - \lambda_x X_t)\frac{1-e^{-\lambda_X}}{\lambda_X} = 0$)

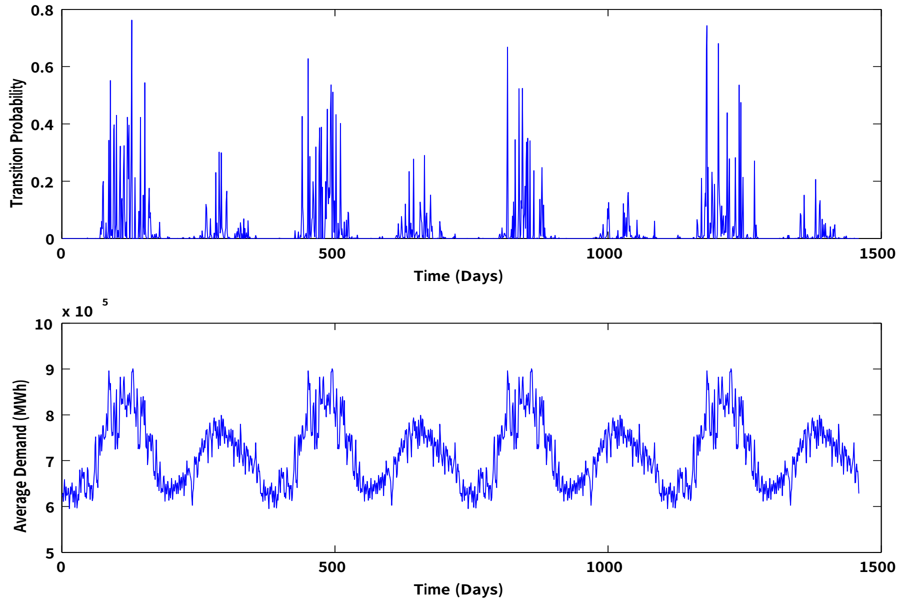

Figure 7. Transition Probabilities π_{NS} from a Non-spike Regime to a Spike Regime (Transition threshold value $D = z - s$)

and Figure 8, respectively, assuming that the time horizon for the calculation is four years. While these figures are one typical path of 50 time simulations, we can easily notice that the transition probabilities π_{NS} and π_{SN} are time-varying and have the seasonality characteristic of being high in summer and winter, which corresponds to the changes in average demand. This result contrasts sharply with the regime-switching models that assume constant transition probabilities.

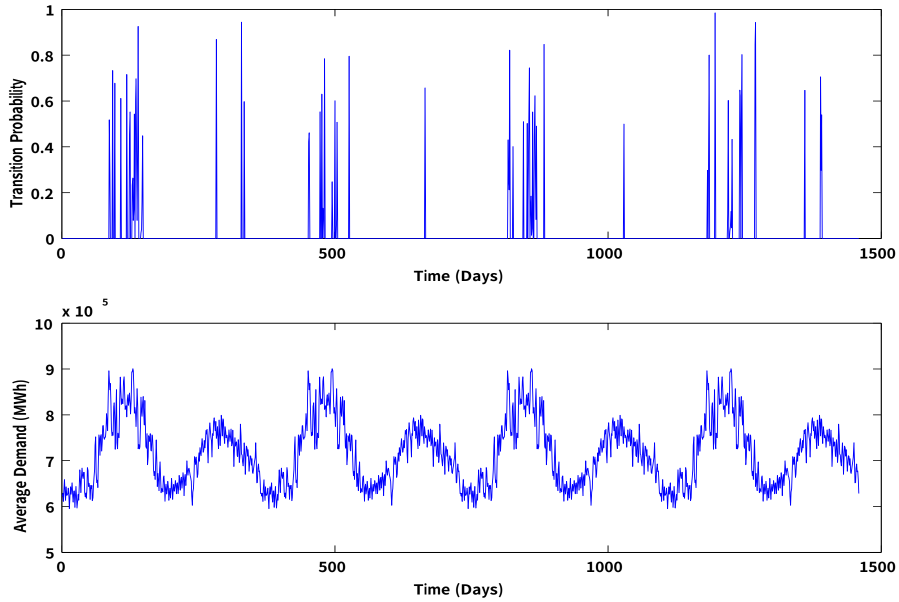

Figure 8. Transition Probabilities π_{SN} from a Spike Regime to a Non-spike Regime (Transition threshold value $D = z - s$)

3.5. Comparison with a Jump Diffusion Model under Non-constant Regime Switching

We compare the transition probabilities obtained above with those in the jump diffusion model of Thompson, Davison, and Rasmussen (2003) that does not incorporate the weekly and annual trend of spot prices. Their spot price model is given by the expressions

$$dP = .4(15\sin(\frac{2\pi t - 15.4\pi}{24}) + 27 - P)dt + .2Pdx + (J_1 - P)dq_1 + (J_2 - P)dq_2 \qquad (17)$$

where $J_1 \in N(700, 100)$, $J_2 \in N(100, 10)$, dq_1 is a Poisson process whose intensity is $\lambda_{\mathrm{up}}(P)$, and dq_2 is a Poisson process whose intensity is $\lambda_{\mathrm{down}}(P)$. Here, the intensity of upper jumps and lower jumps depends on the price level. When the price is less than $\$100/MWh$, $\lambda_{\mathrm{up}}(P)$ and $\lambda_{\mathrm{down}}(P)$ are assumed to be $.0001P$ and 0, respectively. On the other hand, they are otherwise $.01P$ and 0.85, respectively. Note that we calculate the daily transition probabilities for the jump diffusion model in order to adjust the time intervals for both models.

We show the sample path of prices for four years and the transition probabilities of their jump diffusion model in Figure 9. Note that while many price spikes are observed in Figure

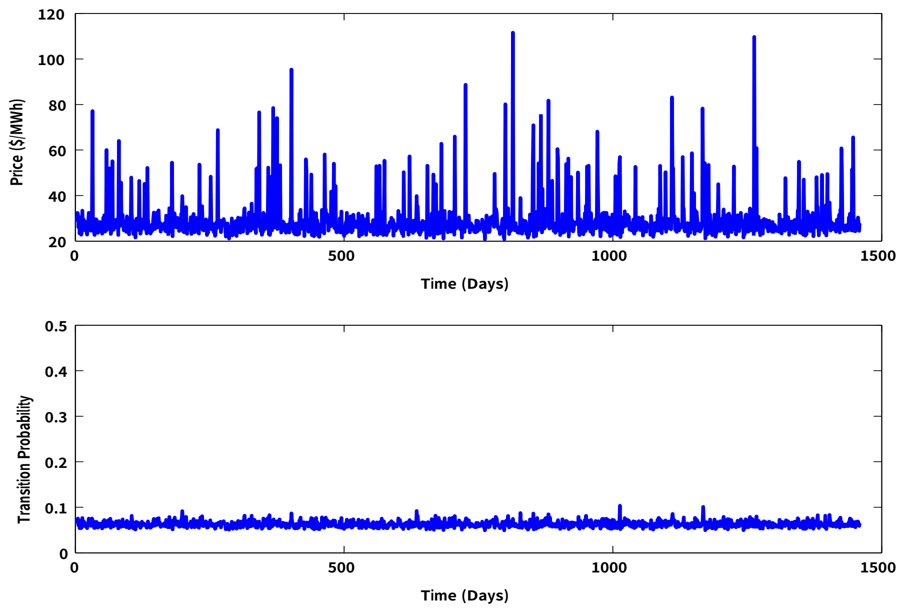

Figure 9. Prices and the Transition Probabilities from a Non-spike Regime to a Spike Regime for $\lambda_{\text{up}}(P) = 0.0001P \ (P \leq \$100/MWh)$ and $0.01 \ (P \geq \$100/MWh)$ Generated by a Jump Diffusion Model

9, the transition probabilities shown in Figure 9 are almost constant throughout the simulation period. This result stands in contrast to that of the structural model described in Figure 7 and Figure 8, and does not fit well the seasonality characteristic of price spikes tending to occur in summer and winter when the demand level is high.

Since Figure 9 shows the less time-varying transition probabilities and a larger number of spikes than are observed in reality, we consider the following two cases: In the first case, the upward jump intensity $\lambda_{\text{up}}(P)$ is taken to be $0.0005P$ when the price is less than $\$100/MWh$, and 0.05 when the price is greater than $\$100/MWh$. In the other case, $\lambda_{\text{up}}(P)$ is taken to be $0.00002P$ when the price is less than $\$100/MWh$, and 0.002 when the price is greater than $\$100/MWh$. The results are illustrated in Figure 10 and Figure 11. Compared with Figure 9, Figure 10 successfully generates time-varying transition probabilities, although there are too

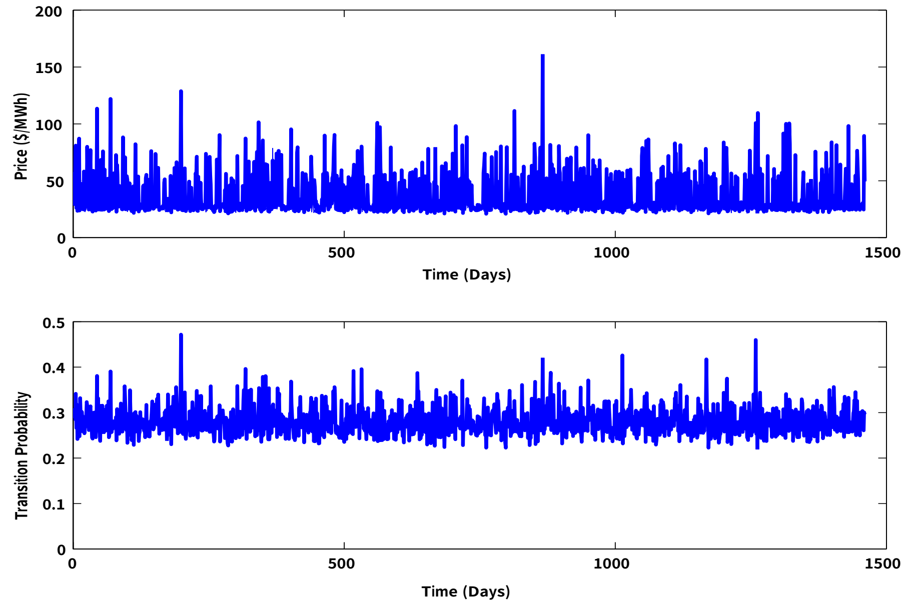

Figure 10. Prices and the Transition Probabilities from a Non-spike Regime to a Spike Regime for $\lambda_{\text{up}}(P) = 0.0005P \;\; (P \leq \$100/MWh)$ and $0.05 \;\; (P \geq \$100/MWh)$

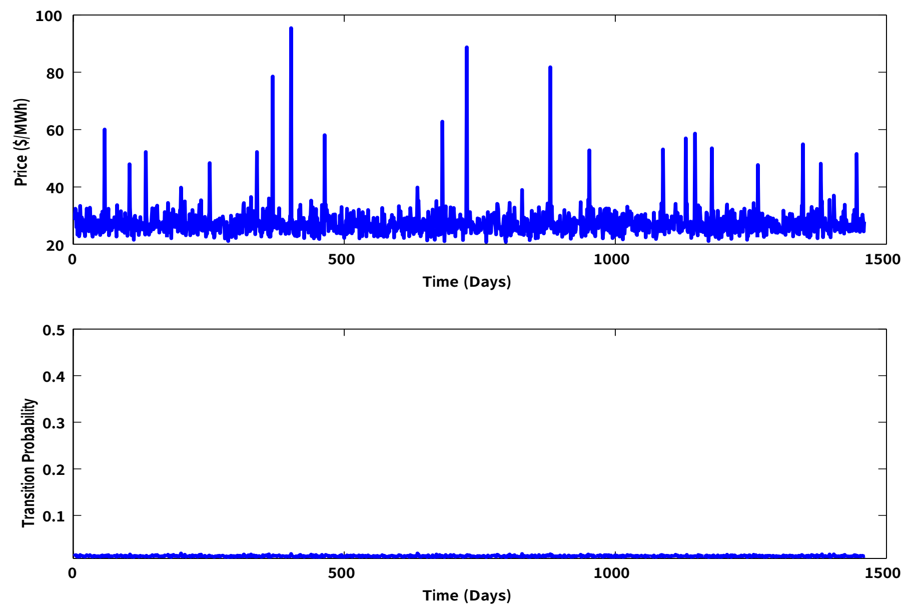

Figure 11. Prices and the Transition Probabilities from a Non-spike Regime to a Spike Regime for $\lambda_{\text{up}}(P) = 0.00002P \;\; (P \leq \$100/MWh)$ and $0.002 \;\; (P \geq \$100/MWh)$

many price spikes. On the other hand, Figure 11 successfully generates an appropriate number of price spikes, while the corresponding transition probabilities are almost constant. These experiments tell us that even if the upper jump probability is adjusted, it seems difficult for the jump diffusion model to generate an appropriate number of price spikes and time-varying transition probabilities at the same time.

As discussed by Lucia and Schwartz (2002), it is important to incorporate seasonality to acurately model electricity prices. The results here suggest that the structural model may be promising in that it can capture the seasonality of not only the price spikes, but also the transition probabilities of regime switching much more easily than current jump diffusion models.

4. Conclusions and Further Discussions

We have described the transition probabilities of regime switching in electricity prices based on the supply and demand of electricity using the structural model of Kanamura and Ōhashi (2004). We have shown that the transition probabilities are not constant, but depend on the current demand level, the deterministic trend of demand change, and the trend caused by the deviation of temporary demand fluctuation from its long-term mean. We have calibrated the model to historical data from the PJM market, and empirically obtained the seasonality of the transition probabilities of regime switching. This contrasts with most current regime-switching models of electricity prices that assume constant transition probabilities. The results obtained here are consistent with the observed characteristics of price spikes in electricity markets where the spikes tend to occur in summer and winter when the demand level is high.

These results support the argument by Lucia and Schwartz (2002) that incorporating seasonality is important in modeling electricity prices. Non-constant, time-varying transition probabilities should be utilized for accurate risk management of electricity prices. This issue will be examined in our future research.

Appendix A. Proof of Proposition 1

Since X_t is a mean-reverting process in equation (2), we have

$$X_s = e^{-\lambda_X(s-t)}X_t + \frac{\mu_X}{\lambda_X}(1 - e^{-\lambda_X(s-t)}) + \sigma \int_t^s e^{-\lambda_X(s-u)}dW_u. \tag{A1}$$

Therefore the distribution of demand at time $t+1$ is given by $D_{t+1} \sim N(\mu_D, \sigma_D^2)$. The transition probabilities from a non-spike regime to a spike regime is the summation of probabilities of demand from $z-s$ to ∞ under the condition that at time t demand is less than $z-s$. The opposite transition probabilities are also obtained in the same way. $\|$

Appendix B. Effect of Changing the Threshold Value

In order to measure the effect of changing the regime boundary, we calculate the transition probabilities of π_{NS} and π_{SN} by setting $D = z$. The results are illustrated in Figure 12 and 13, respectively. Comparing

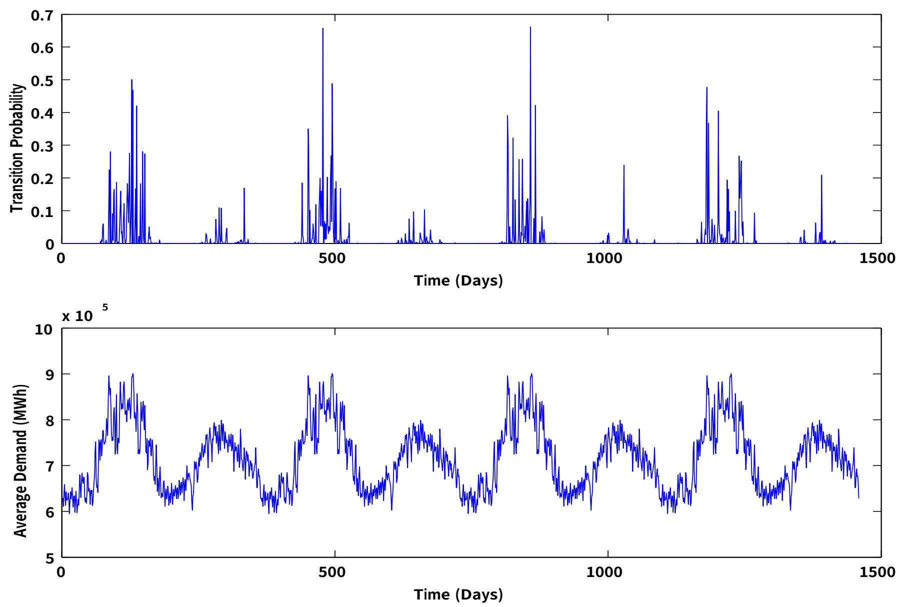

Figure 12. Transition Probabilities from a Non-spike Regime to a Spike Regime (π_{NS}): Transition Point $D = z$

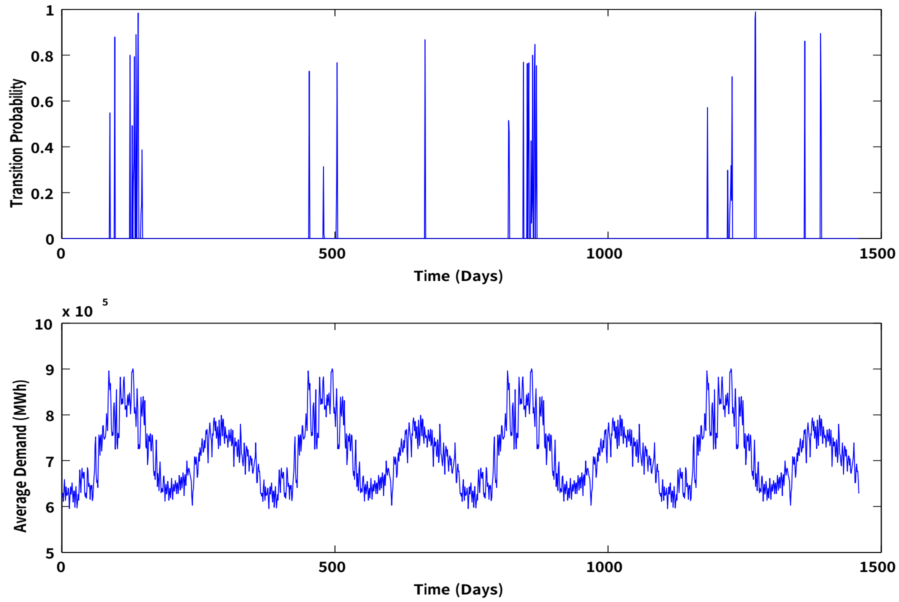

Figure 13. Transition Probabilities from a Spike Regime to a Non-spike Regime (π_{SN}): Transition Point $D = z$

$D = z$ with $D = z - s$, the transition probabilities with $D = z$ from the non-spike regime to the spike regime (π_{NS}) are less than those with $D = z - s$, as in Figure 7 and 12. In the case of those from the spike regime to the non-spike regime, the results are also the same. In both cases, however, the results are qualitatively similar to those with the threshold value $D = z - s$, and the transition probabilities depend on demand and are not constant.

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Notes

[1]Thompson, Davison, and Rasmussen (2003) divide two regimes according to electricity price levels and thus do not assume constant transition probabilities.

[2]This assumption does not apply to several real electricity markets in which market manipulation by power companies is a serious problem. However, in some markets, such as the Nordpool market and the PJM market, where power companies seem to behave competitively, this assumption seems a good first approximation.

[3]The definition of the regimes is similar to that in Thompson, Davison, and Rasmussen (2003). Also, the non-spike regime corresponds to the standard (mean-reverting) regime and the spike regime to the spike (or jump) regime, respectively, in Jong and Huisman (2002)

[4]One may change $z-s$ to z or $z+s$ and obtain a qualitatively same result. See Appendix B.