ARMA modeling in practice

Model Specification (Identification)

Parameter Estimation

Diagnostics

USE OF THE MODEL

Model Specification (Identification)

1) Plot the series

2) Perform **transformations** to achieve stationarity (if necessary)

3) Compute sample **ACF** and **PACF**

4) Make a guess on **p** and **q** and go to next stage (Estimation)

Sample ACF

Given an observed time series $(X_1, X_2, \ldots, X_t, X_{t+1}, \ldots, X_T)$ with sample mean $\hat{m}$, the sample autocorrelation of order **j** is defined as the ratio between the sample autocovariance and the sample variance:

$$\hat{r}(j) = \frac{\sum_{t=j+1}^{T} (X_t - \hat{m})(X_{t-j} - \hat{m})}{\sum_{t=1}^{T} (X_t - \hat{m})^2}$$

Partial Autocorrelation Function (PACF)

With the sample ACF alone it is difficult to discriminate among AR(p) processes.

For example AR(2) vs. AR(3) or AR(3) vs. AR(4). $\Longrightarrow$ **PACF**

In order to introduce the partial autocorrelation function we start with a very simple AR(1) example: $X_t = f \; X_{t-1} + a_t$

In the AR(1) model, X_{t-1} has all the information relevant to explain X_t. Once we have X_{t-1} we do not care about X_{t-2} ,X_{t-3}, X_{t-4}, …

If we run a linear regression of X_t on X_{t-1} the OLS coefficient it will converge f

If we run a linear regression of X_t on X_{t-2} the OLS coefficient it will converge f^2

However if we run a linear regression of X_t on X_{t-1} **and** X_{t-2} the OLS coefficient of X_{t-2} will converge to 0 as X_{t-1} *"does all the job"* in predicting X_t

The correlation coefficient of X_{t-2} and X_t (net of X_{t-1}) is zero.

The **partial correlation coefficient of order p (f_{pp})** is defined as the coefficient of X_{t-p} in the linear regression of X_t on $X_{t-1}, X_{t-2}, \ldots X_{t-p}$.

$$X_t = f_{11} X_{t-1} + e_t$$

$$X_t = f_{12} X_{t-1} + f_{22} X_{t-2} + e_t$$

$$X_t = f_{13} X_{t-1} + f_{23} X_{t-2} + f_{33} X_{t-3} + e_t$$

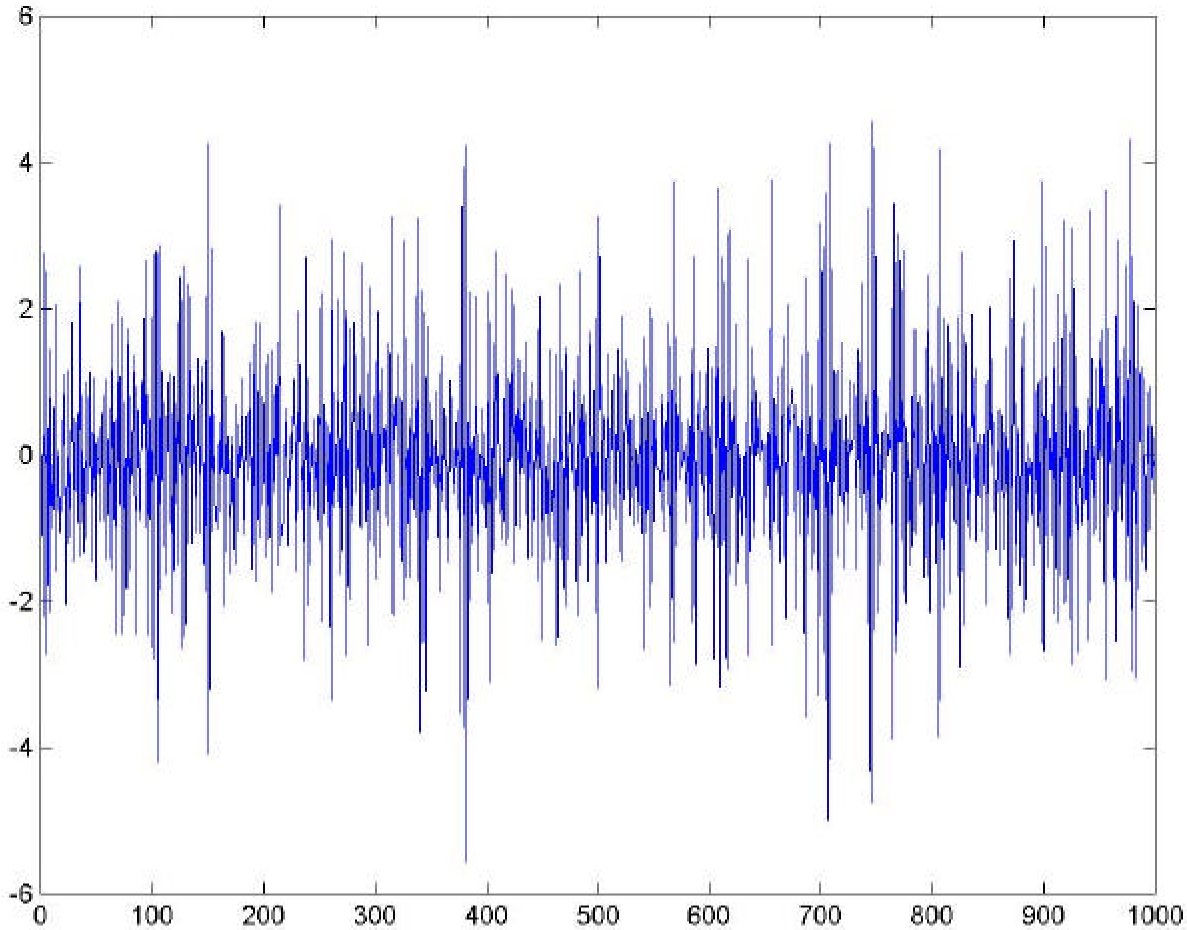
$$\vdots$$

$$X_t = f_{1p} X_{t-1} + f_{2p} X_{t-2} + \ldots + f_{pp} X_{t-p} + e_t$$

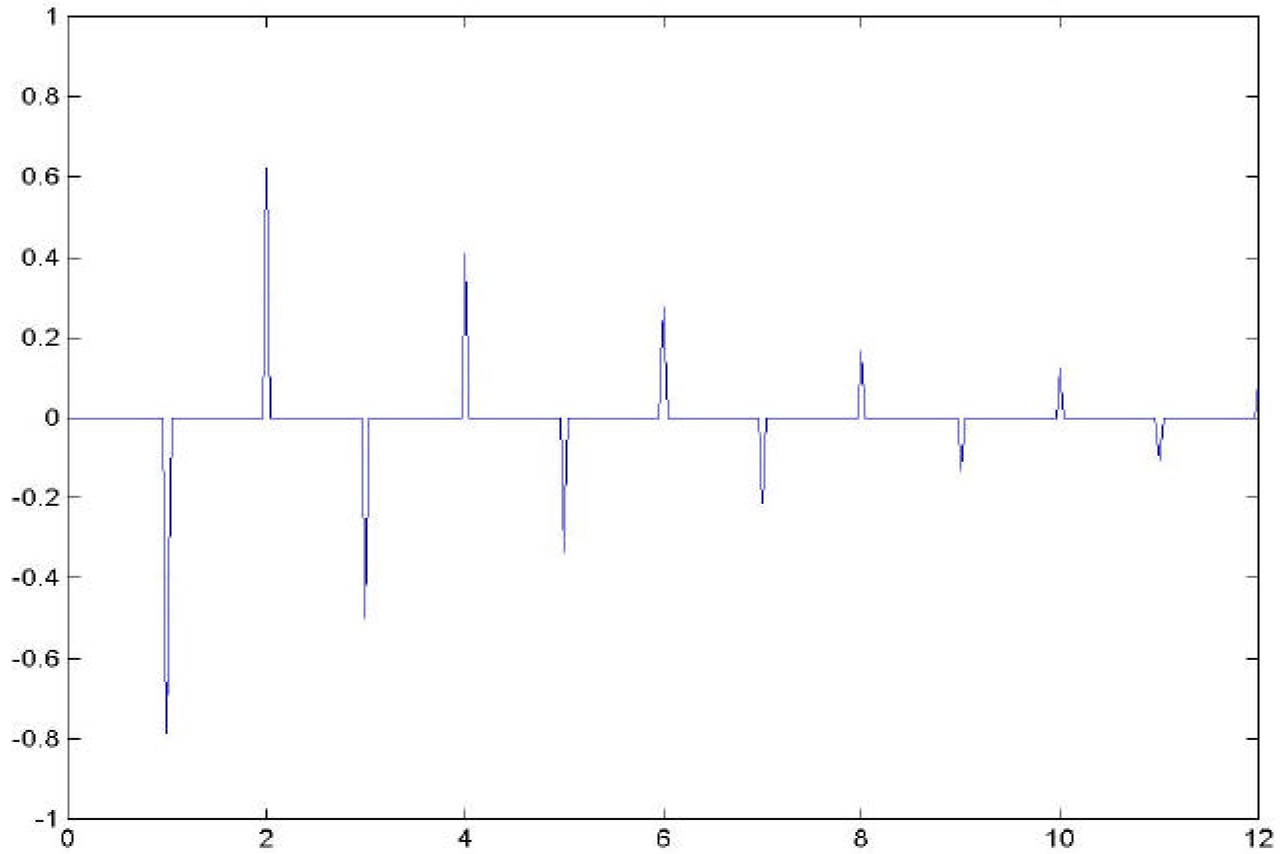
In AR(p) processes only the first p partial autocorrelations are different from 0.

Since a MA(q) can be written as AR(∞), the partial correlations of MA models converge to 0 in the limit.

Simulated AR(1): ϕ = -0.8

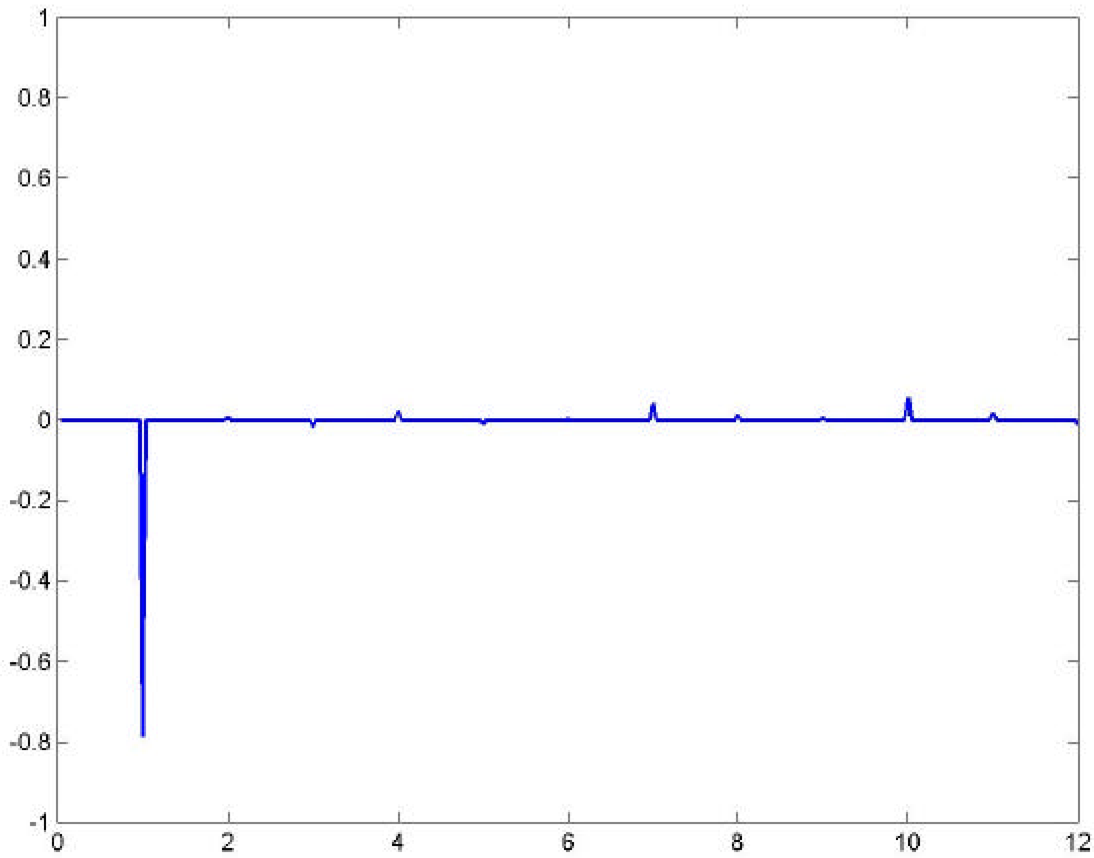


Simulated AR(1): f **= -0.8** Sample ACF T=1000



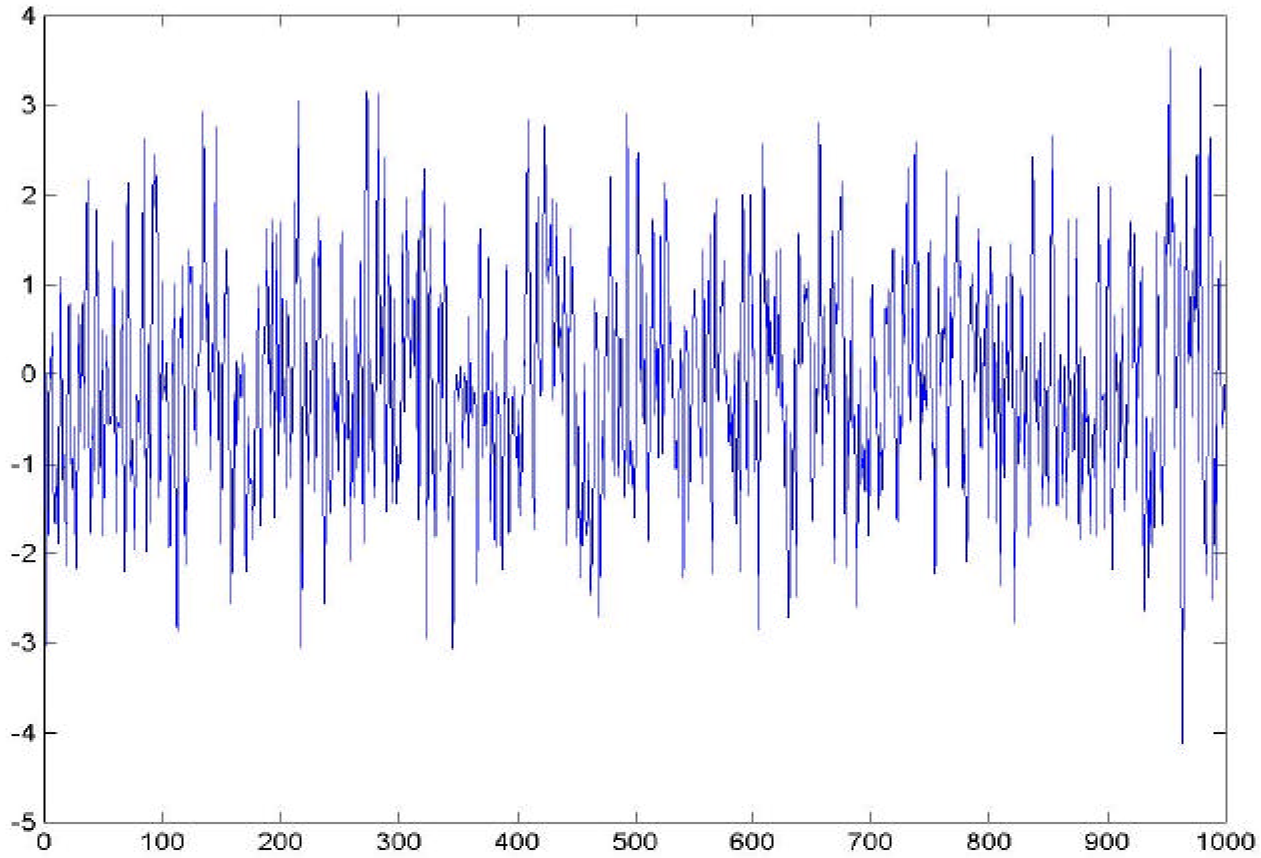
-0.7881
0.6223
-0.4988
0.4081
-0.3370
0.2771
-0.2124
0.1668
-0.1305
0.1228
-0.1062
0.0838

Simulated AR(1): ϕ **= -0.8** Sample PACF T=1000

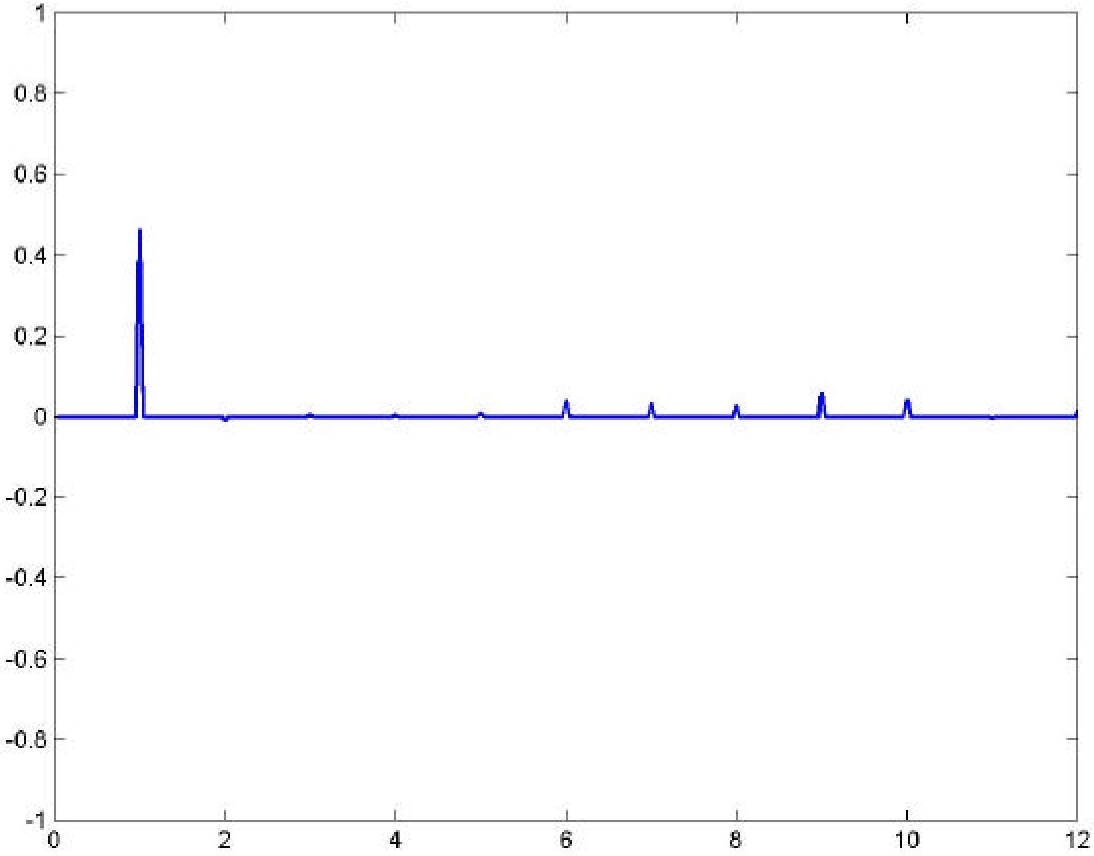


-0.7876
0.0052
-0.0176
0.0245
-0.0077
0.0011
0.0417
0.0120
0.0049
0.0558
0.0180
-0.0155

Simulated MA(1): θ = 0.7

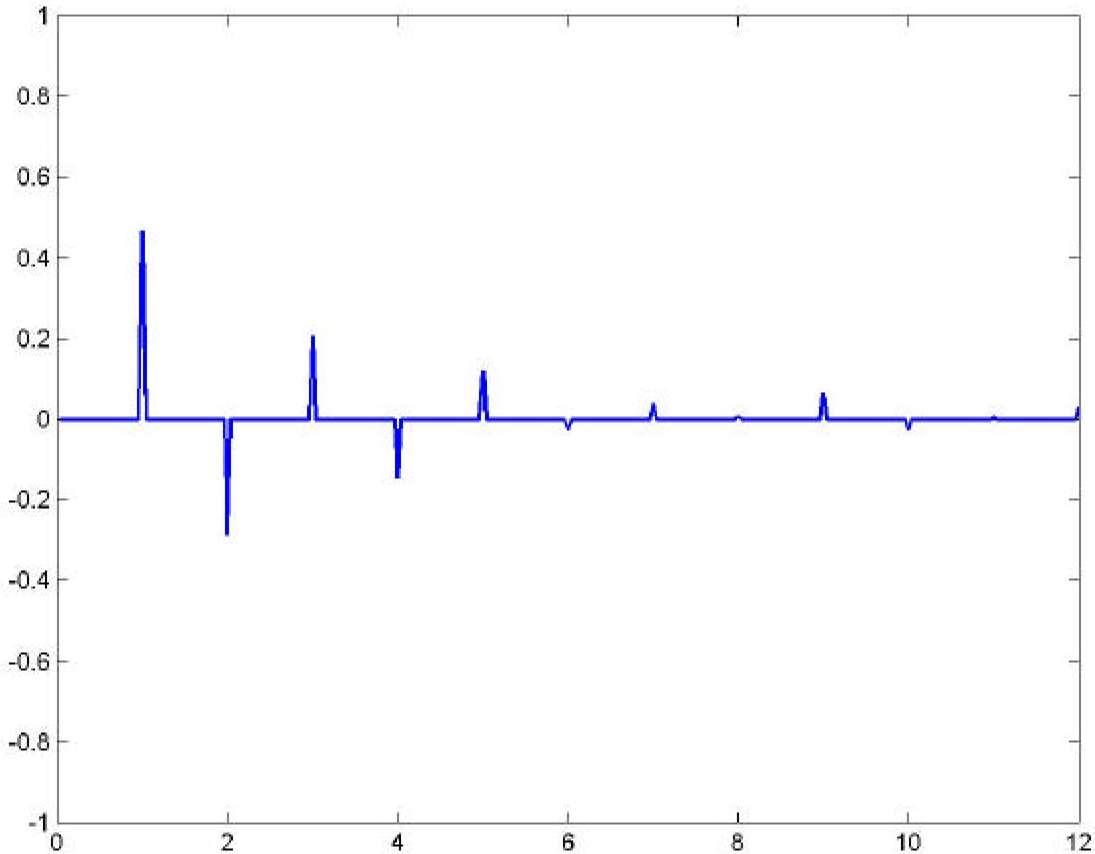


Simulated MA(1): θ = 0.7 Sample ACF T=1000



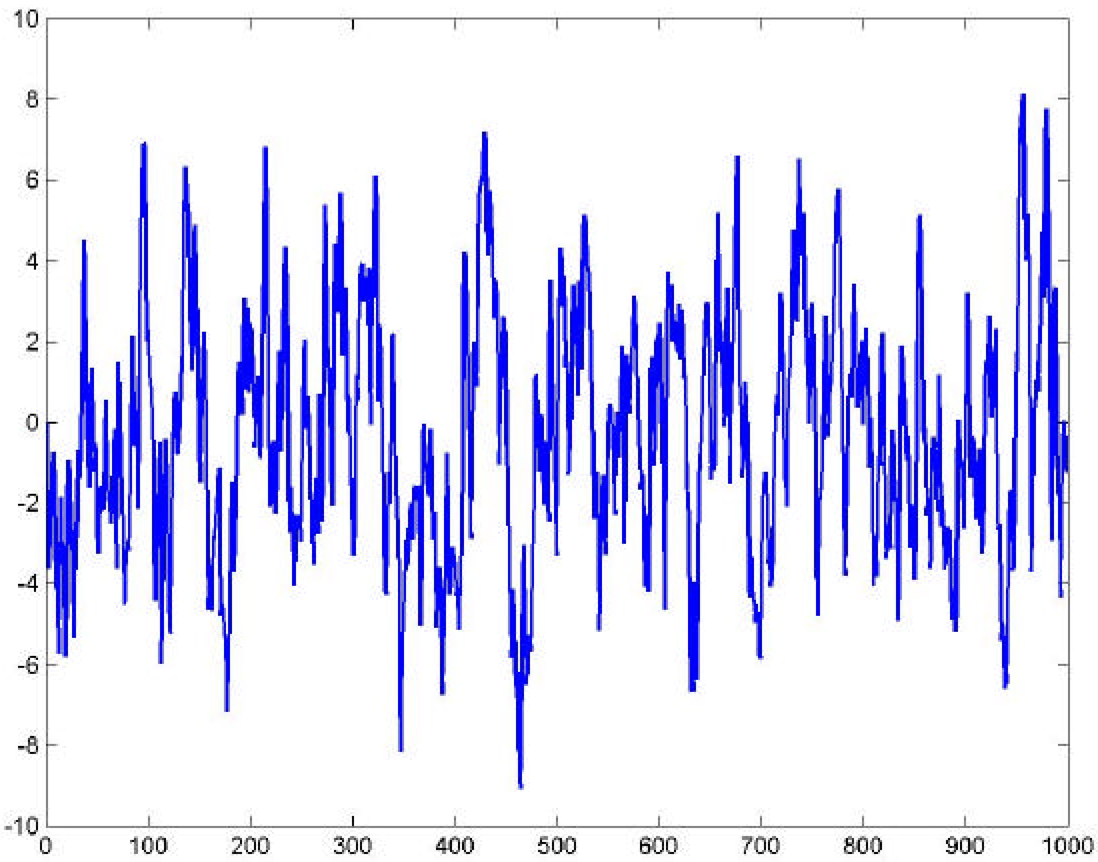
0.4653
-0.0094
0.0062
0.0036
0.0070
0.0417
0.0354
0.0282
0.0607
0.0433
-0.0044
0.0144

Simulated MA(1): q = 0.7 Sample PACF T=1000

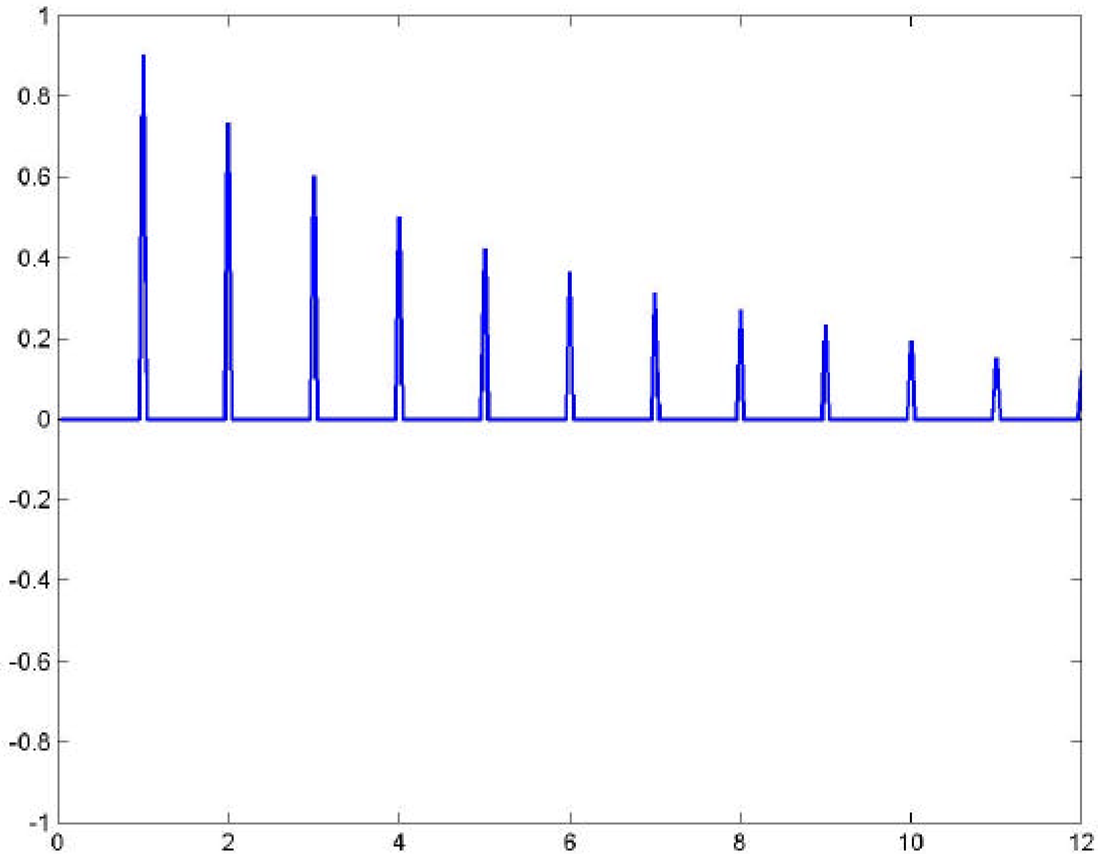


0.4676
-0.2865
0.2067
-0.1436
0.1190
-0.0279
0.0394
0.0066
0.0650
-0.0274
0.0038
0.0300

Simulated ARMA(1): f = 0.8 q = 0.9

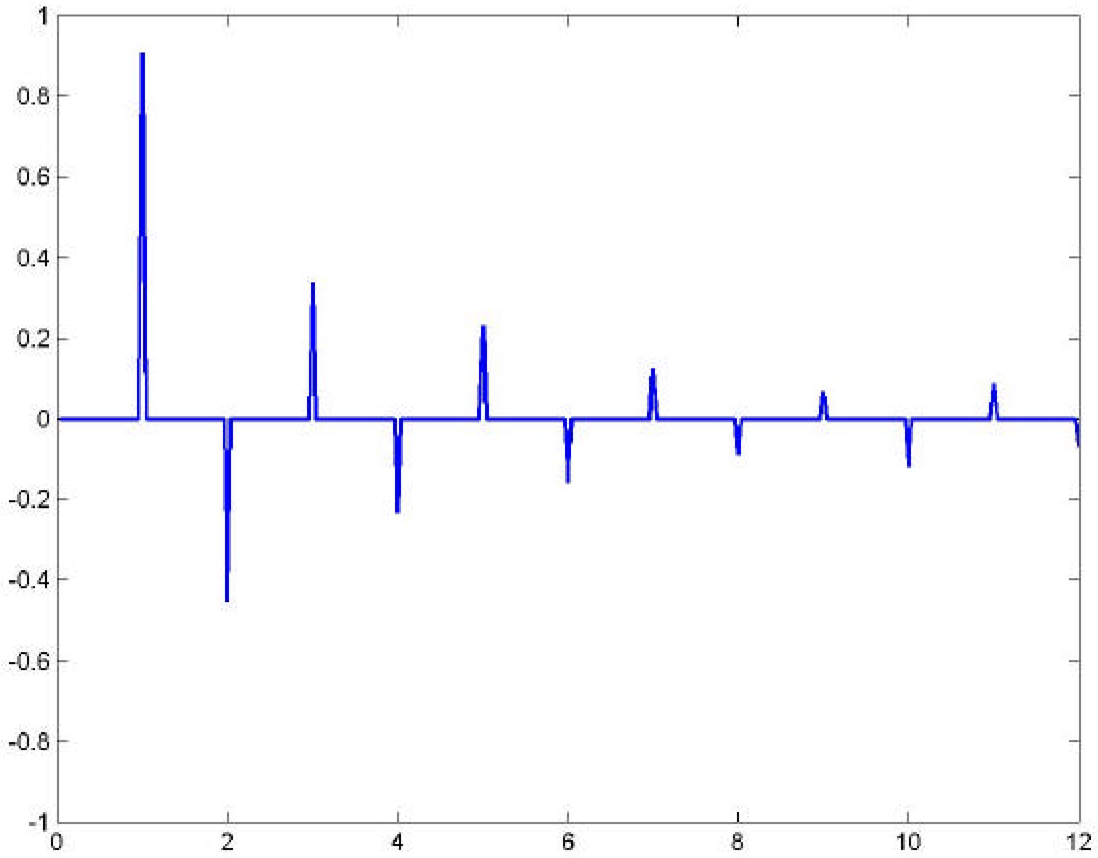


Simulated ARMA(1): f = 0.8 q = 0.9 Sample ACF T=1000



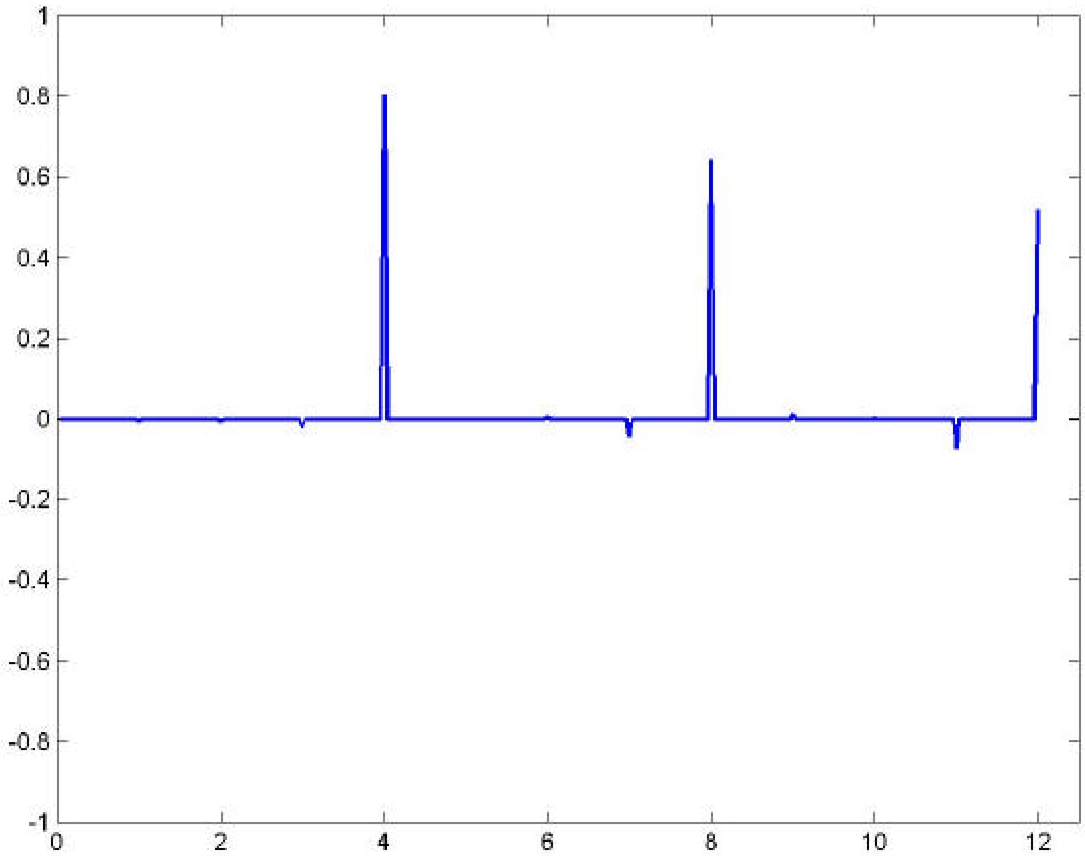
0.9040
0.7344
0.6037
0.5018
0.4241
0.3656
0.3152
0.2717
0.2347
0.1946
0.1537
0.1215

Simulated ARMA(1): ϕ **= 0.8** θ **= 0.9** Sample PACF T=1000



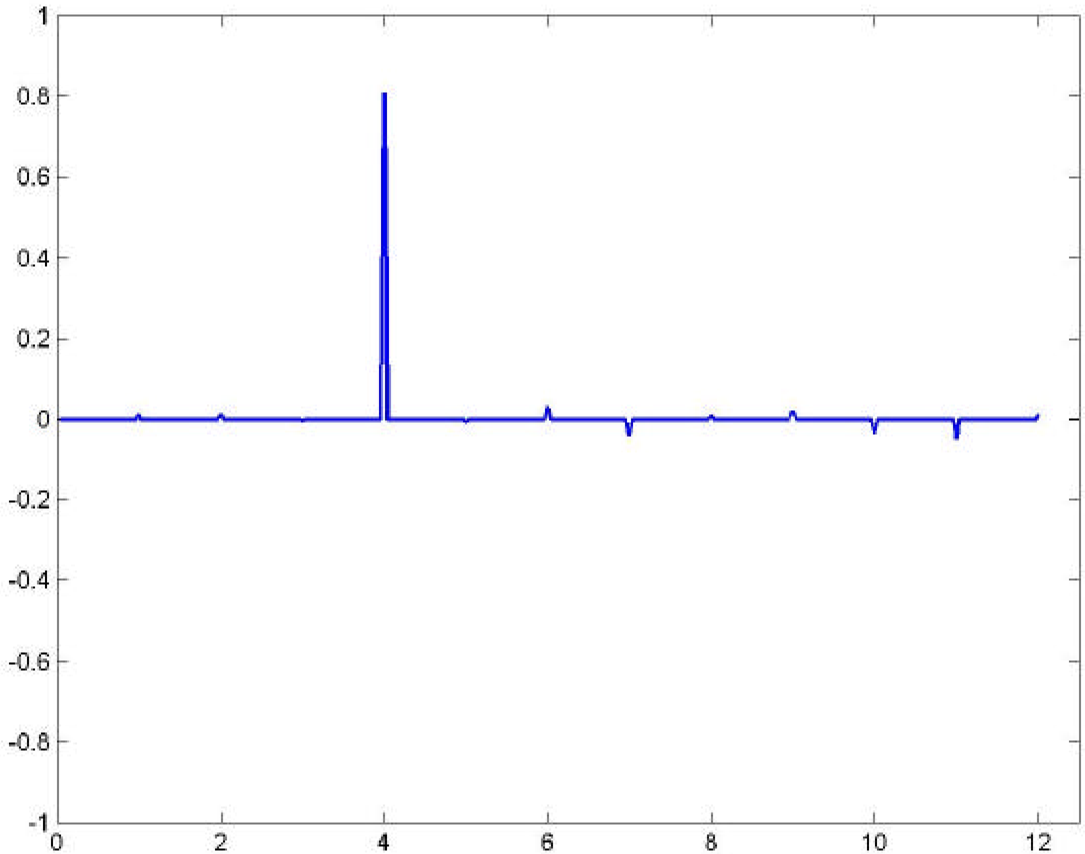
0.9061
-0.4523
0.3383
-0.2310
0.2323
-0.1596
0.1268
-0.0874
0.0685
-0.1187
0.0862
-0.0672

Simulated seasonal AR(1)$_{4}$: f_4 = 0.8 Sample ACF T=1000



-0.0067
-0.0061
-0.0207
0.8020
-0.0007
0.0058
-0.0434
0.6430
0.0102
0.0011
-0.0742
0.5183

Simulated seasonal $AR(1)_4$: $\phi_4 = 0.8$ Sample PACF T=1000



0.0117
0.0122
-0.0022
0.8080
-0.0075
0.0304
-0.0402
0.0079
0.0209
-0.0360
-0.0498
0.0109