HEDGING VOLATILITY RISK

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Abstract

Volatility risk plays an important role in the management of portfolios of derivative assets as well as portfolios of basic assets. This risk could have been managed more efficiently using options on volatility that were proposed in the past but were never offered for trading mainly due to the lack of a cost efficient tradable underlying asset.

The objective of this paper is to introduce a new volatility instrument, an option on a straddle, which can be used to hedge volatility risk. The design and valuation of such an instrument are the basic ingredients of a successful financial product. Unlike the proposed volatility index option, the underlying of this proposed contract is a traded at-the-money-forward straddle, which should be more appealing to potential participants. In order to value these options, we combine the approaches of compound options and stochastic volatility. Our numerical results show that the straddle option price is very sensitive to the changes in volatility which means that the proposed contract is indeed a very powerful instrument to hedge volatility risk.

I. Introduction

Risk management is concerned with various aspects of risk, in particular, price risk and volatility risk. While there are various efficient instruments (and strategies) to deal with price risk, exhibited by the volatility of asset prices, there is practically only one instrument which deals with volatility risk, namely "volatility swaps", which is basically a forward contract on realized volatility[1]. In this paper we are introducing a new volatility instrument, an option on a **forward-start straddle**, which in our opinion dominates the usefulness of existing alternatives, including "volatility swaps".

While option traders, in general, are subject to volatility risk, as well as other risks, the main concern of delta-neutral volatility traders is the risk that volatility may change. Though many players may be betting on the direction that volatility may take in the future and would not protect their downside risk, some may seek to hedge their bets at least against large movements in volatility[2]. It is true that one can bet on volatility changes (or hedge them) with a strategy that combines holding of static options, all the out-of-the money ones, and dynamically trade the underlying asset (see Carr and Madan (1998)). Such a strategy, however, may be very costly and not practical for most users.

Given the large and frequent shifts in volatility in the recent past[3] especially in periods like the summer of '97, the fall of '98 and the fall of 2001, there is a growing need for instruments to hedge volatility risk. Past proposals of such instruments included futures and options on a volatility index[4]. The idea of developing a volatility index was suggested by Brenner and Galai (1989) and (1993)[5]. In 1993 the Chicago Board Options Exchange (CBOE) has introduced a volatility index, named VIX, which was based on implied volatilities from options on the SP100 index[6]. No options, or futures, were offered on this index. The main reason, in our opinion, that such derivatives were not introduced is the lack of a cost-efficient tradable underlying asset which market makers could use to hedge their positions and to price them.

The first theoretical paper[7] to value options on a volatility index is by Grunbichler and Longstaff (1996). They specify a mean reverting square root diffusion process for volatility. Their framework is similar to that of Hull and White (1987), Stein and Stein (1991) and others. Since volatility is not trading they assume that the premium for volatility risk is proportional to the level of volatility. This approach is in the spirit of the equilibrium approach of Cox, Ingersoll and Ross (1985) and Longstaff and Schwartz (1992). A more recent paper by Detemple and Osakwe (2000) also uses a general equilibrium framework to price European and American style volatility options. They emphasize the mean-reverting in log volatility model.

Recently, the CBOE has changed the methodology that was used to calculate VIX. The new forward-looking volatility index uses current option prices to predict the next 30 days realized volatility. In essence, the volatility index uses the S&P500 at-the-money put and call and all out-of-the-money options weighted by the inverse of the square of their strike prices. This approach is based on the work by Derman et al. (1996) and Carr and Madan (1998). Demeterfi et al. (1999) provide a detailed description of volatility and variance swaps. Following the introduction of futures the CBOE is now planning to introduce options on the new VIX. The exchange argues that derivatives on this index will be an efficient tool to trade volatility. Since, however, the underlying asset for these derivatives is a combination of all available options with different strikes it will not be a practical replicating portfolio.

Rather than an option on an implied volatility index or an option on an index computed from the prices of many options, some of which hardly trade, we propose an option on a straddle (STO). The key feature of the straddle option is that the underlying asset is an at-the-money-forward (ATMF) straddle. The ATMF straddle is a traded asset priced in the market place and well understood by market participants[8]. Since it is ATMF, its relative value (call + put)/stock is mainly affected by volatility. Changes in volatility translate almost linearly into changes in the value of the underlying, the ATMF straddle[9]. Thus options on the ATMF straddle are options on volatility. We believe that such an instrument will be attractive to market participants, especially to market makers.

Since the CBOE is planning to introduce options on the new VIX it is interesting to compare the new VIX with the volatility index computed from our ATMF straddle using the same stochastic volatility parameters. In Figure II we present graphically the two indices. Though VIX is less linear than NST (our Normalized Straddle) the values are very close. This implies that traders in VIX options should consider hedging with the ATM straddle which seems to be the best possible alternative.

An additional benefit of such an innovation is that it will provide a market price for volatility risk. A few recent papers have examined this issue empirically (e.g., Coval and Shumway (2001), Bakshi and Kapadia (2003), Buraschi and Jackwerth (1999)). Examining different strategies they conclude that the volatility risk premium is negative. Currently there is no market that calibrates this premium as, for example, the risk premium in the stock market. The return on the underlying of the proposed product, an always ATMF straddle, will provide such a calibration.

In the next section we describe in detail the design of the instrument. In section III we derive the value of such an option. Section IV provides the conclusions.

II. The Design of the Straddle Option

One natural group of users of these options are volatility speculators who buy and sell volatility using standard call and put options which are affected by changes in the underlying asset and by interest rates in addition to changes in volatility. It's a package which they may not be interested in and, as expected, is more expensive than a direct bet on volatility. The other potential groups of users are hedgers who mainly trade in the options market, like market makers in options, and portfolio managers who allocate funds between stocks and bonds using a mean-variance analysis. Since their allocation, and performance, may be affected by an unexpected change in volatility they may want to insure against volatility risk. Again, this can be done simply using standard straddles but this approach is inefficient since it insures against both: volatility changes (*vega*) and changes in delta (*gamma*). The price of the straddle reflects the broader coverage which is not sought after. To isolate the volatility risk one could dynamically trade the straddle such that it always is ATMF but such a strategy entails transactions costs that become very high depending on the frequency of rebalancing which in turn depends on volatility itself. Thus, the desired instrument, proposed next, would be a hedge against volatility risk **only** and should cost less than the alternatives, including transactions costs.

To manage the market volatility risk, say of the S&P500 index, we propose a new instrument, a straddle option or STO (K_{STO}, T_1, T_2) with the following specifications. At the maturity date T_1 of this contract, the buyer has the *option* to buy a then at-the-money-forward straddle with a pre specified exercise price K_{STO}. The buyer receives both, a call and a put, with a strike price equal to the forward price, given the index level at time T_1. The straddle matures at time T_2.

Our proposed contract has two main features: first, the value of the contract at maturity depends on the volatility expected in the interval T_1 to T_2 and therefore it is a tool to hedge volatility changes. It is sensitive to volatility but not to interest rates or changes in the spot. Second, the underlying asset is a traded straddle[10]. We believe that, unlike the volatility options, this design will greatly enhance its acceptance and use by the investment community. Compared to the available alternatives it is the most cost effective.

The proposed instrument is conceptually related to two known exotic option contracts: compound options and forward start options[11]. Unlike the conventional compound option our proposed option is an option on a straddle with a strike price, unknown at time 0, to be set at time T_1 to the forward value of the index level. In general, in valuing compound options it is assumed that volatility is constant (see, for example, Geske (1979)). Given that the objective of the instrument proposed here is to manage volatility risk, we need to introduce stochastic volatility.

III. Valuation of the Straddle Option

In this section we first value the straddle option (STO) assuming deterministic volatility as our benchmark case. We then apply a specific stochastic volatility (SV) model to value the straddle option and illustrate its properties.

A. The Case of Deterministic Volatility

We first analyze the case where volatility changes only once and is known at time zero. We assume a constant volatility σ_1 between time 0 and T_1 (expiry date of STO) and a constant volatility σ_2 between T_1 and T_2 (maturity date of the straddle ST). Given its compound option feature, the derivation of time 0 value of STO involves four steps: Value of the underlying straddle ST at its maturity T_2, $ST(T_2)$, and next at T_1, $ST(T_1)$. And then payoff of STO at its expiry T_1, $STO(T_1)$, and finally at time 0, $STO(0)$.

The payoff of straddle ST at its maturity T_2 is:

$$ST(T_2) = call(T_2) + put(T_2) = |S(T_2) - K_{ST}| \tag{1}$$

where $K_{ST} = S(T_1)e^{r(T_2 - T_1)}$ and $S(T)$ is the stock price at T.

Assuming that the call and put in the straddle are European as is the typical index option and that the Black-Scholes assumptions hold, and using Brenner and Subrahmanyam (1988) approximation for ATM options, we have

$$ST(T_1) \equiv \alpha \cdot S(T_1) = 2(2N(d_1) - 1)S(T_1) \approx \frac{2S(T_1)}{\sqrt{2\pi}}\sigma_2\sqrt{T_2 - T_1}\ , \tag{2}$$

where $d_1 = \frac{1}{2}\sigma_2\sqrt{T_2 - T_1}$. The straddle is practically linear in volatility. The relative value of the straddle, $\alpha = ST(T_1)/S(T_1)$ is solely determined by volatility to expiration.

The payoff of the straddle option (STO) at its expiration T_1 is

$$STO(T_1) = \max\{ST(T_1) - K_{STO}, 0\} = \max\{\alpha \cdot S(T_1) - K_{STO}\} \tag{3}$$

Thus, the price of the STO at any time t, $0 \le t < T_1$ is, using the B-S model:

$$STO_t = \alpha \cdot S_t \cdot N(d) - K_{STO} e^{-r(T_1 - t)} \cdot N(d - \sigma_1 \sqrt{T_1 - t}) \ , \tag{4}$$

where $N(x)$ is cumulative normal distribution function and

$$d = \frac{\ln(\alpha S_t / K_{STO}) + (r + \frac{1}{2}\sigma_1^2)(T_1 - t)}{\sigma_1 \sqrt{T_1 - t}} \ .$$

The sensitivity of STO to the volatility in the first period T_1, called $Vega_1$, is

$$Vega_1 = \frac{\partial STO_t}{\partial \sigma_1} = S_t \sqrt{T_1 - t} \cdot N'(d) \,, \tag{5}$$

where $N'(d)$ is the normal density function, which is a standard result for any option except that d is also determined by α which is in turn determined by σ_2. Thus, *Vega* in the first period is affected by volatility in the second period which makes sense since the payoff at expiration of STO is determined by the volatility in the subsequent period.

The sensitivity of STO to the volatility in the second period $T_2 - T_1$, called $Vega_2$, is given by

$$Vega_2 = \frac{\partial STO_t}{\partial \sigma_2} = S_t N(d) \frac{\partial \alpha}{\partial \sigma_2} = S_t N(d) \cdot 2\sqrt{T_2 - T_1} \cdot N'(d_1) \,. \tag{6}$$

Therefore, $Vega_2$ is also a function of the volatility in the current period, not just the volatility of the subsequent period.

$Vega_1$ and $Vega_2$ are proportional to the square root of the length of each period. Each of them depends on the volatility in both periods but primarily on the volatility in its own period. Thus, $Vega_1$ could be smaller or larger than $Vega_2$ depending on the volatility in each period.[12]

B. The Case of Stochastic Volatility

We now turn to the case which is the very reason for offering a straddle option, the stochastic volatility case. We assume a risk-neutral diffusion process and a stochastic volatility (SV) model similar to the one by Stein and Stein (1991):

$$dS_t = rS_t dt + \sigma_t S_t dB_t^1, \tag{7}$$

$$d\sigma_t = \delta(\theta - \sigma_t)dt + k dB_t^2. \tag{8}$$

Equation (7) describes the dynamics of an equity index S_t with a stochastic volatility σ_t. Equation (8) describes the dynamics of volatility itself which is reverting to a long run mean θ where δ is the adjustment rate and k is the volatility of volatility. B_t^1 and B_t^2 are two independent Brownian motions. r is the risk-free rate.

The conditional probability density function of S_T is given by

$$f(S_T \mid S_t, \sigma_t; r, T-t, \delta, \theta, k) = e^{-r(T-t)} f_0(S_T e^{-r(T-t)}) \tag{9}$$

where

$$f_0(S_T) = \frac{1}{2\pi}\left(\frac{S_t}{S_T}\right)^{3/2} \frac{1}{S_t}\int_{-\infty}^{\infty} I\left(\left(\eta^2 + \frac{1}{4}\right)\frac{T-t}{2}\right)\cos\left(\eta \ln\frac{S_T}{S_t}\right)d\eta \tag{9a}$$

in which the function $I(\lambda)$ is given by equation (8) of Stein and Stein (1991)[13].

The transition probability density function of σ_T is normal with mean $\theta + (\sigma_t - \theta)e^{-\delta(T-t)}$ and variance $k^2(1-e^{-2\delta(T-t)})/2\delta$,

$$f(\sigma_T \mid \sigma_t; T-t, \delta, \theta, k) = \frac{1}{\sqrt{\frac{\pi k^2}{\delta}(1-e^{-2\delta(T-t)})}} \exp\left[-\frac{\left(\sigma_T - \theta - (\sigma_t - \theta)e^{-\delta(T-t)}\right)^2}{\frac{k^2}{\delta}(1-e^{-2\delta(T-t)})}\right]. \tag{10}$$

The joint distribution of S_T and σ_T is

$$f(S_T,\sigma_T)=f(S_T)f(\sigma_T) \tag{11}$$

since the two Brownian motions are assumed independent.

Using risk-neutral valuation with the above joint distribution, the value of the straddle ST at time T_1 is

$$\begin{aligned} ST_{T_1} &= 2e^{-r(T_2-T_1)}\int_{S_{T_1}e^{r(T_2-T_1)}}^{\infty}(S_{T_2}-S_{T_1}e^{r(T_2-T_1)})f(S_{T_2}\mid S_{T_1})dS_{T_2} \\ &\equiv 2S_{T_1}F(\sigma_{T_1};T_2-T_1,r,\delta,\theta,k) \end{aligned} \tag{12}$$

where the strike price K_{ST} is $S_{T1}e^{r(T_2-T_1)}$, and function $F(\sigma_{T_1})$ is defined by the second equal sign.

Given the values of the straddle ST, the price of the straddle option, STO, at time $t=0$ can be computed as

$$STO_0=\int_0^{\infty}G(\sigma_{T_1})f(\sigma_{T_1}\mid\sigma_0)d\sigma_{T_1} \tag{13}$$

where

$$G(\sigma_{T_1})=2F(\sigma_{T_1})e^{-rT_1}\int_{\frac{K_{STO}}{2F(\sigma_{T_1})}}^{\infty}\left(S_{T_1}-\frac{K_{STO}}{2F(\sigma_{T_1})}\right)f(S_{T_1}\mid S_0)dS_{T_1} \tag{14}$$

The values of STO are computed numerically and presented in Table 1a and in Figures 1a to 1d using a range of parameter values. Next to the values from the SV model, in 1a, we present the values using the BS model (*k*=0). As expected, the value of this compound option using the SV model is larger than the value of this option using the

BS model. The difference between the two depends on the values of the other parameters in the SV model and the strike price K_{STO}. For relatively low strike prices, K_{STO}, the effect of stochastic volatility is rather small and the values are not that different from a BS value, ignoring stochastic volatility. For higher strike prices (out of the money) the effect of k, the volatility of volatility, is much larger. For K_{STO}=12, slightly out-of-the-money, the value of STO at *k*=.3 is about 1.6 times larger than STO at *k*=.1 (1.22 vs. 0.47) while the BS value is only 0.36. Figure 1b shows the effect of initial volatility, σ_0. At low strike prices an increase in initial volatility has a small effect on the values of STO. At high strike prices the value of STO is lower but the marginal effect of σ_0 is much higher. Figure 1c shows the effect of θ, the long-run volatility on STO. For low values of θ, the value of STO is declining as we get to the ATM strike. Hedging against changes in volatility in a low volatility environment is not worth much. Figure 1d shows the combined effect of volatility and *k*, volatility of volatility, at the ATM strike of STO. As expected, the value of STO increases in both and is rather monotonic. Stochastic volatility has a relatively bigger effect in a low volatility environment.

The effect of the various parameters on the value of STO could be discerned from the previous table and graphs but a better understanding of the complex relationships can be obtained from an examination of the various sensitivities given in Figures 2a and 2b. Figure 2a provides the sensitivity of STO to changes in volatility, which is the main issue here. Figure 2a displays these values at 5 levels of initial σ_0. The values are high at all levels of initial volatility, though they tend to decline as volatility increases, indicating that changes in volatility could be effectively hedged by the straddle option. It becomes less effective as the strike price K_{STO} increases, the option is out-of-the-money. Figure 2b displays values for the sensitivity of STO to *k*, volatility of volatility. The higher is k, the higher is the "*vega*" of STO. It is most sensitive at intermediate values of the strike price and approaches zero as the strike price increases. We also find, the graph is not presented here, that the sensitivity with respect to the time to maturity of the straddle itself, $T_2 - T_1$, is higher for a maturity of 3 months than for a longer maturity, 6 months or a year, because the incremental value of STO at a shorter maturity is larger than at a longer maturity where the value is already high.

An interesting observation regarding the value of STO emerges. Does STO have a higher value, relative to BS value, in markets with higher volatility? It seems that higher σ, for a given *k* (volatility of volatility), tends to reduce the differences between SV values and BS values since σ is the dominant factor in the valuation. However, if higher σ is accompanied by higher *k*, STO values will be served little by a stochastic volatility model.

IV. Conclusions

The stochastic behavior of volatility, which has always affected options premiums, has been, for the most part, ignored by most participants. However, any risk management system must cope with volatility risk and it can do so in several ways, using existing instruments and/or a dynamic strategy. In this paper we propose a derivative instrument, an option on a straddle that can be used to hedge the risk inherent in stochastic volatility. This option could be traded on exchanges and used for risk management. It compares favorably with other possible alternatives; it is sensitive only to volatility, the underlying asset is tradable and it is the most cost effective instrument.

Since valuation is an integral part of using and trading such an option we derive the value of such an option using a stochastic volatility model. We compare the value of such an option to a benchmark value given by the BS model. We find that the value of such an option is very sensitive to changes in volatility and therefore cannot be approximated by the BS model.

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Figure I
S&P 500 Volatility Index (VIX)

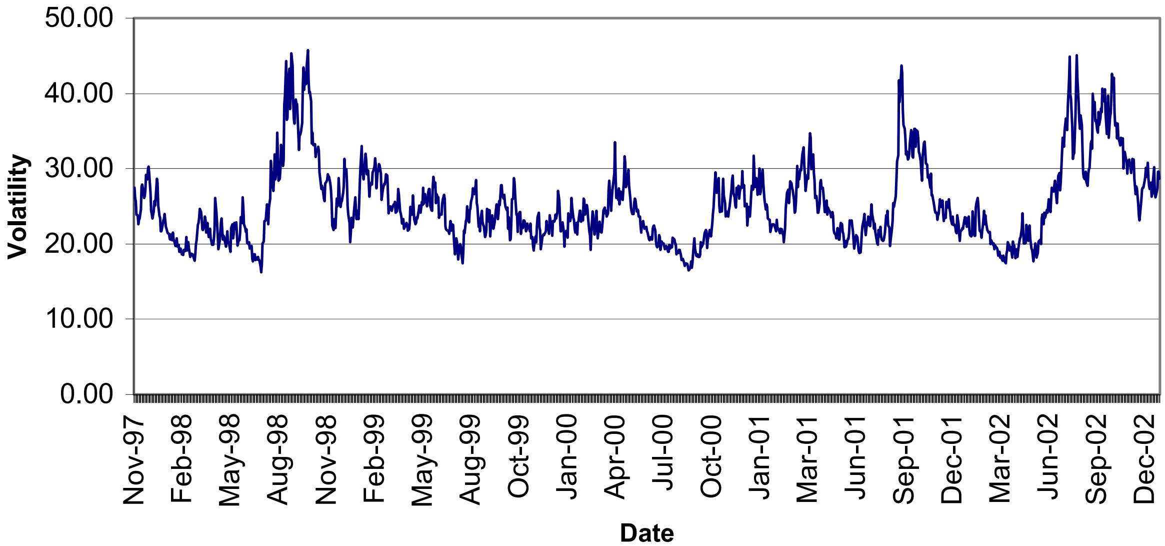


Figure I Closing level on the S&P 500 Volatility Index (VIX). The sample period is November 1, 1997 – December 31, 2002. Source: CBOE.

FIGURE II
Volatility Index based on ATMF straddle (NST) and the new VIX for different values of initial volatility at a high level of stochastic volatility

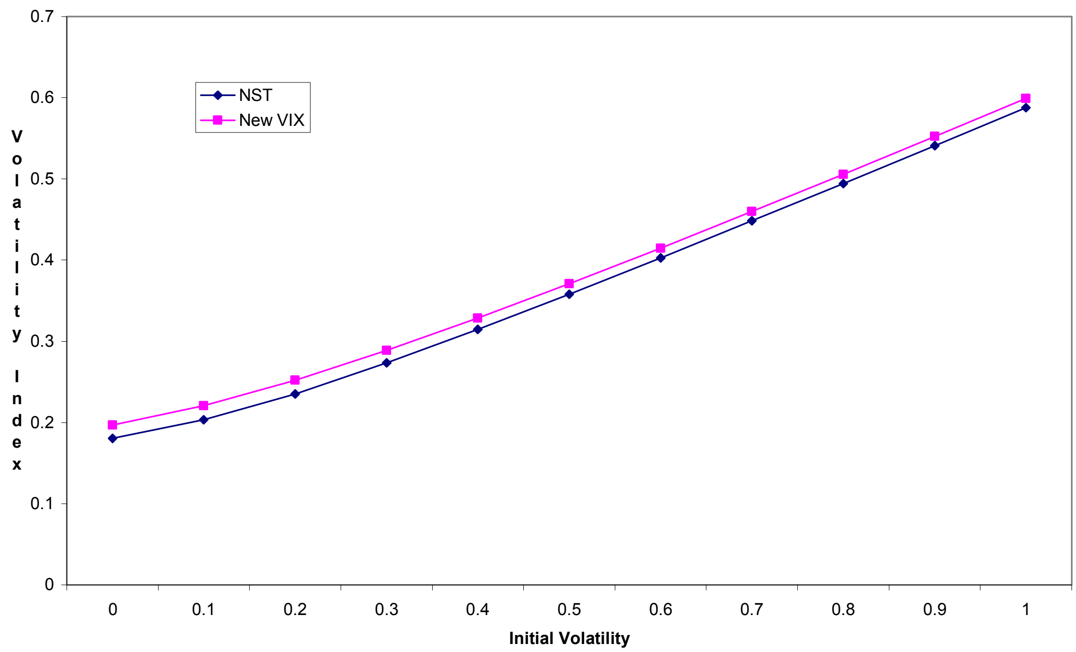


Table 1a: The value of the Straddle Option, *STO,* at t = 0 for a combination of strike price K_{STO} and volatility of volatility k. S_0 =100, r = 0, σ_0 *,intial volatility,*= 0.20, θ*, long-run volatility* = 0.20, δ*, reversion parameter* = 4.00, T_1 =0.5, T_2 = 1.0.

k / K_{STO}	0 (BS)	0.10	0.20	0.30	0.40	0.50
0	11.274	11.352	11.580	11.841	12.146	12.564
1	10.274	10.352	10.583	10.874	11.231	11.699
2	9.274	9.352	9.585	9.904	10.311	10.829
3	8.274	8.352	8.587	8.933	9.388	9.957
4	7.274	7.352	7.590	7.962	8.465	9.083
5	6.274	6.352	6.592	6.990	7.542	8.210
6	5.274	5.352	5.594	6.020	6.619	7.338
7	4.274	4.352	4.601	5.054	5.700	6.467
8	3.277	3.360	3.629	4.111	4.793	5.602
9	2.308	2.408	2.713	3.222	3.919	4.754
10	1.439	1.564	1.907	2.428	3.113	3.942
11	0.774	0.908	1.254	1.757	2.406	3.195
12	0.355	0.470	0.771	1.223	1.812	2.538
13	0.140	0.218	0.446	0.820	1.331	1.981
14	0.048	0.092	0.245	0.531	0.957	1.521
15	0.014	0.035	0.129	0.335	0.674	1.152
16	0.004	0.013	0.065	0.206	0.466	0.861
17	0.001	0.004	0.032	0.124	0.318	0.636
18	0.000	0.001	0.016	0.074	0.215	0.466
19	0.000	0.000	0.007	0.044	0.144	0.339
20	0.000	0.000	0.003	0.026	0.096	0.245

FIGURE 1a
The value of *STO* for a combination of strike prices K_{STO} and volatility of volatility k

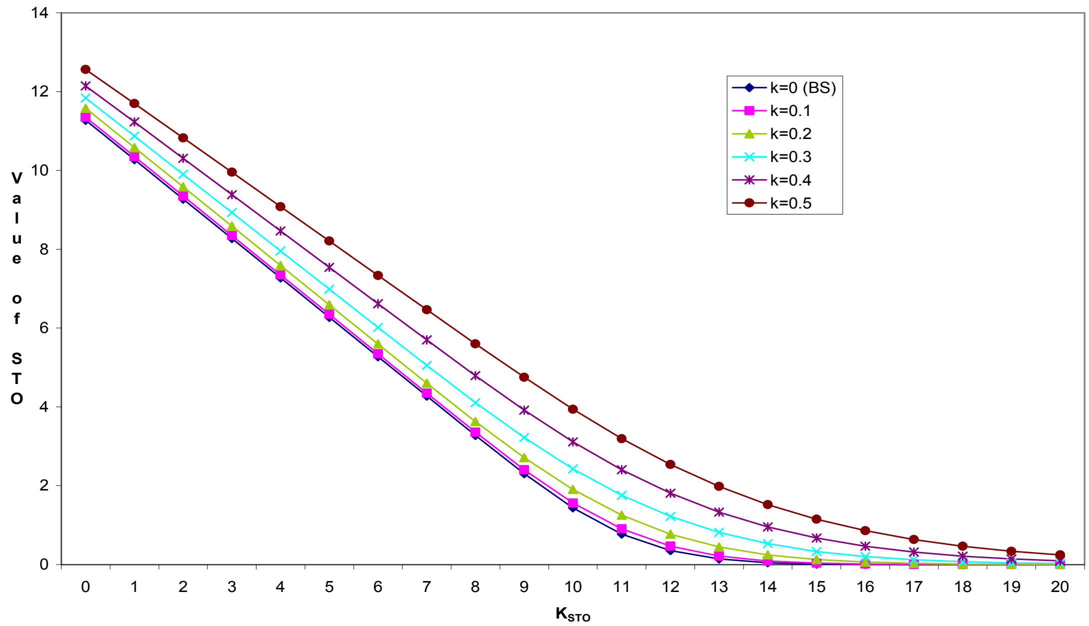


FIGURE 1b
The value of *STO* for a combination of strike price K_{STO} and initial volatility

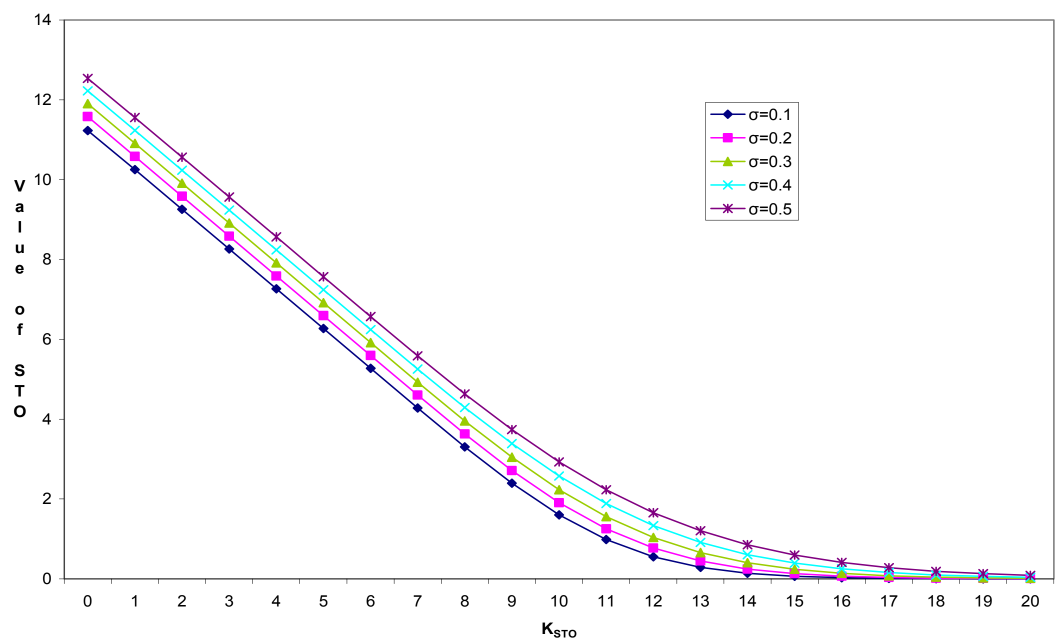


FIGURE 1c

The value of *STO* for a combination of K_{STO} and the long-term level of volatility

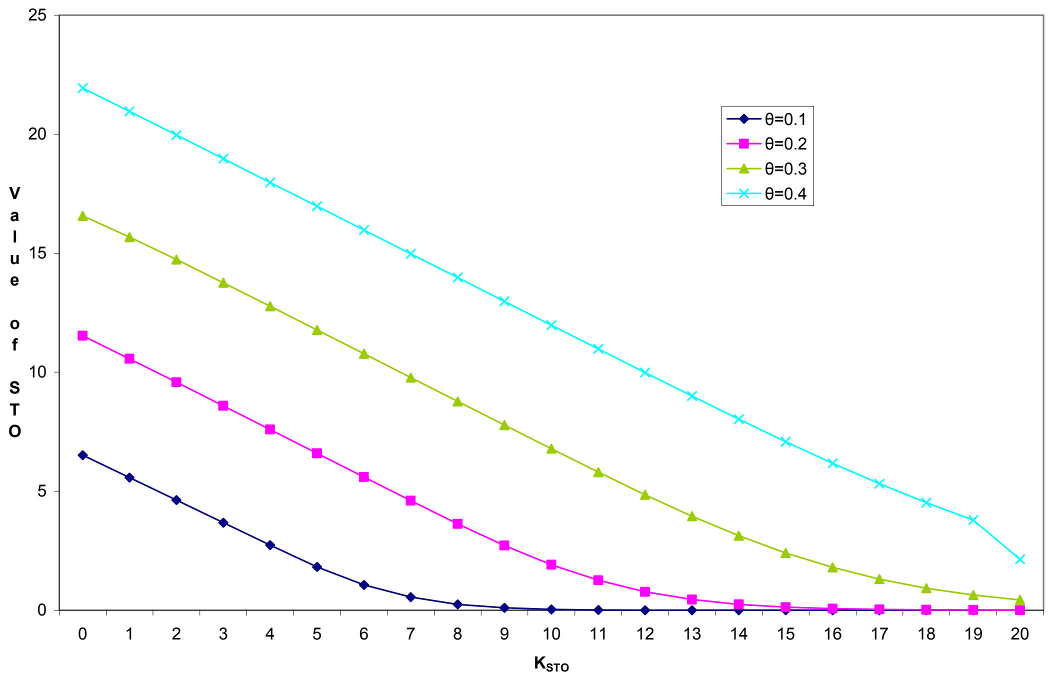


FIGURE 1d

The value of *STO* for different combinations of initial volatility and volatility of volatility

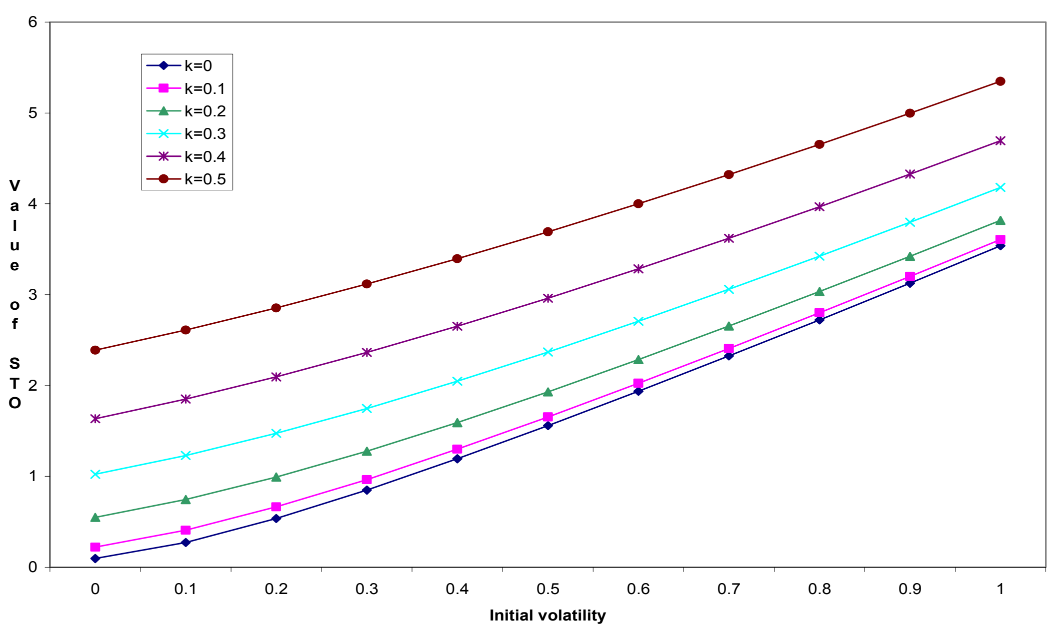


FIGURE 2a
The sensitivity of *STO* to K_{STO} at different levels of initial vol.

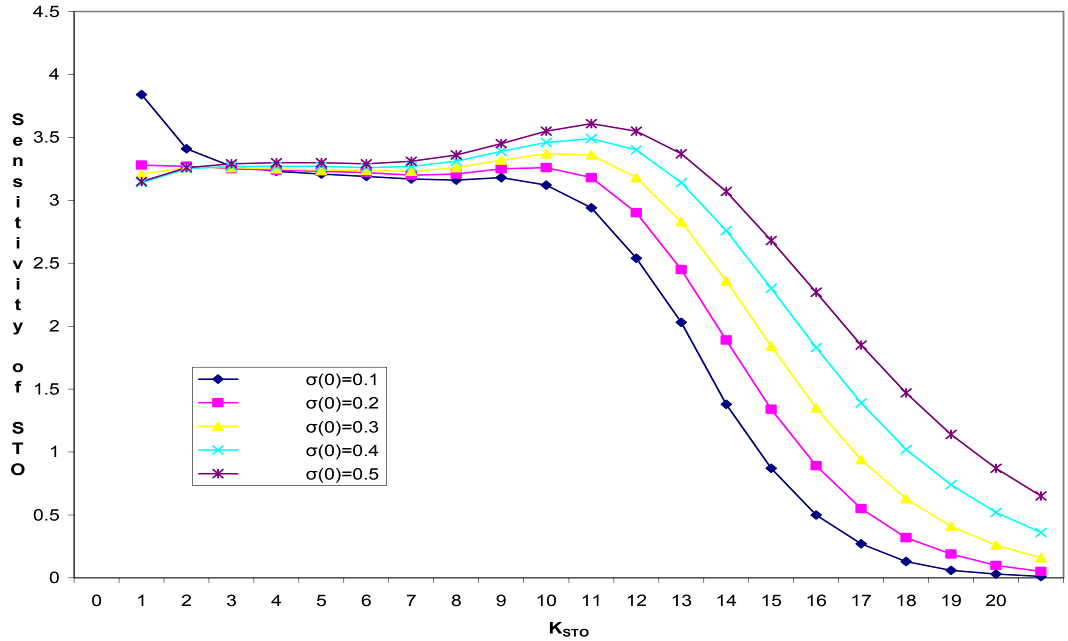
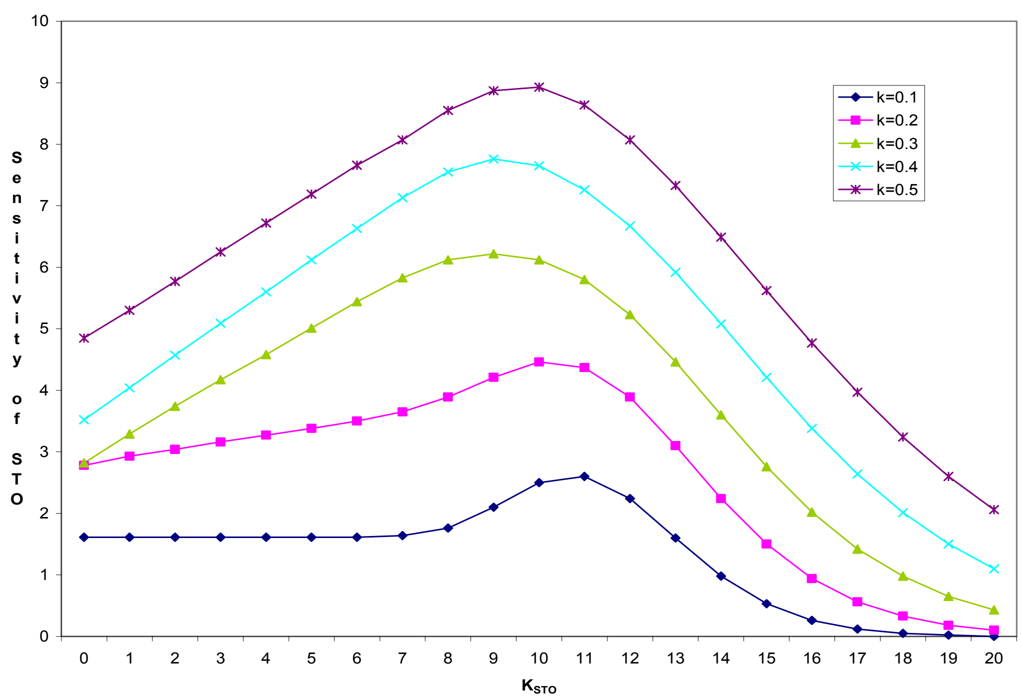


Figure 2b
The Sensitivity of *STO* to K_{STO} at different levels of k



Endnotes

1 A comprehensive analysis of volatility and variance swaps is provided in Demeterfi, Derman, Kamal and Zou (1999).

2 One of the strategies used by Long-Term-Capital-Management (LTCM) was to sell volatility on the S&P 500 index and other European indices (see Lowenstein R. (2000), p. 123).

3 The volatility of volatility can be observed from the behavior of a volatility index, VIX, provided in Figure 1. Though different, the new VIX exhibits a very similar behavior.

4 There were several attempts to introduce volatility derivatives (e.g., the German DTB launched a futures contract on the DAX volatility index) but those attempts were largely unsuccessful.

5 Gastineau (1977) and Galai (1979) have proposed an index of option prices which corresponds to some implied volatility index. Such an index is also described in Cox and Rubinstein (1985).

6 Brenner and Galai (1993) have introduced a volatility index based on implied volatilities from at-the-money options of two near term maturities. The old VIX used a similar methodology.

7 Brenner and Galai (1993) use a binomial framework to value such options where tradability is assumed implicitly.

8 ATMF straddles are traded mainly in the FX market and are quoted on a volatility basis.

9 Strictly speaking this is true in a B-S world (See Brenner and Subrahmanyam (1988)) but here, with stochastic volatility, it may include other parameters (e.g. vol. of volatility).

10 Theoretically there is no difference if the delivered option is a call, a put or a straddle since they are all ATMF. Practically, however, there may be some difference in prices due, for example, to transactions costs. A straddle would provide a less biased hedge vehicle.

11 Forward start options are paid for now but start at some time T_1 in the future. The instrument proposed here is different from the instrument proposed by Gary Gastineau in one aspect; while in his proposal the number of options the buyer gets is adjusted to reflect the change in the underlying price, there is no such adjustment in our proposal.

12 If the volatilities and the periods happen to be the same then the difference between *Vega₁* and *Vega₂* will be negligible.

13 Equation (8) in Stein and Stein (1991) is

$$I(\lambda) = \exp(L\sigma_0^2/2 + M\sigma_0 + N)$$

where *L*, *M* and *N* are functions of λ, given by

$$L(\lambda) = -A - a\left(\frac{\sinh\left(ak^2\tau\right) + b\cosh\left(ak^2\tau\right)}{\cosh\left(ak^2\tau\right) + b\sinh\left(ak^2\tau\right)}\right),$$

$$M(\lambda) = B\left(\frac{b\sinh\left(ak^2\tau\right) + b^2\cosh\left(ak^2\tau\right) + 1 - b^2}{\cosh\left(ak^2\tau\right) + b\sinh\left(ak^2\tau\right)} - 1\right),$$

$$N(\lambda)=\frac{a-A}{2a^2}\left(a^2-AB^2-B^2a\right)k^2\tau+\frac{B^2(A^2-a^2)}{2a^3}\times\frac{(2A+a)+(2A-a)e^{2ak^2\tau}}{A+a+(a-A)e^{2ak^2\tau}}$$

$$+\frac{2AB^2(a^2-A^2)e^{ak^2\tau}}{a^3\left(A+a+(a-A)e^{2ak^2\tau}\right)}-\frac{1}{2}\ln\left[\frac{1}{2}\left(\frac{A}{a}+1\right)+\frac{1}{2}\left(1-\frac{A}{a}\right)e^{2ak^2\tau}\right],$$

$$a=\sqrt{A^2-2C},\qquad b=-A/a,\qquad \tau=T-t,$$

$$A=-\delta/k^2,\qquad B=\theta\delta/k^2,\qquad C=-\lambda/(k^2\tau).$$