Forward start options

Definitions

Forward start options are options whose strike will be determined at some later date.

Description

Like a standard option, a forward-start option is paid for in the present; however the strike price is not fully determined until an intermediate date before expiration.

Why use a forward start option?

An application of forward start options is employee stock options. This typically has a forward start feature since its strike price is not fixed when the employee begins to work. Rather, at that time, he is promised that he will receive stock options at periodic dates in the future conditional upon his continued employment. The strikes of these options are currently unknown, but will be set to be at-the-money on the subsequent grant dates.

Theory

A forward start option with time to maturity T starts at-the-money or proportionally in- or out-of-the-money after a known elapsed time t in the future. The strike is set equal to a positive constant a times the asset price S after the known time t.

The following holds:

	Call option	**Put option**
$a < 1$	Starts (1- a)% in-the-money	Starts (1- a)% out-of-the-money
$a > 1$	Starts (a -1)% out-of-the-money	Starts (a -1)% in-the-money
$a = 1$	Option starts at-the-money	

The value of a forward start option is then given by

$$c = Se^{(b-r)t}\left[e^{(b-r)(T-t)} \cdot N(d_1) - a\, e^{-r(T-t)} \cdot N(d_2)\right];$$
$$p = Se^{(b-r)t}\left[a\, e^{-r(T-t)} \cdot N(-d_2) - e^{(b-r)(T-t)} \cdot N(-d_1)\right]$$

where

$$d_1 = \frac{\ln\left(\frac{1}{a}\right) + \left(b + \frac{s^2}{2}\right)(T-t)}{s\sqrt{T-t}};$$
$$d_2 = d_1 - s\sqrt{T-t}$$

and $b = \text{riskfree rate} - \text{continuous dividend yield.}$

Example

Consider an employee who receives a call option with forward start 3 months from today. The option starts 10% out-of-the-money, time to maturity is 1 year from today, the stock price is 60, the risk-free interest rate is 8%, the continuous dividend yield is 4%, and the expected volatility of the stock is 30%.

Hence,

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{1}{a}\right) + \left(b + \frac{s^2}{2}\right)(T-t)}{s\sqrt{T-t}} \\ &= \frac{\ln\left(\frac{1}{1.1}\right) + \left((0.08-0.04) + \frac{0.3^2}{2}\right)(1-0.25)}{0.3\sqrt{1-0.25}} \\ &= -0.1215 \end{aligned}$$

$$
\begin{aligned}
d_2 &= d_1 - s\sqrt{T-t} \\
&= -0.1215 - 0.3\sqrt{1-0.25} \\
&= -0.3813
\end{aligned}
$$

$$N(d_1) = 0.4517;\ N(d_2) = 0.3515$$

Now the value of the forward start call option is:

$$
\begin{aligned}
c &= S \cdot e^{(b-r)t}\left[e^{(b-r)(T-t)} \cdot N(d_1) - a\, e^{-r(T-t)} \cdot N(d_2)\right] \\
&= 60 \cdot e^{(0.04-0.08)0.25}\left[e^{[(0.08-0.04)-0.08](1-0.25)} \cdot 0.4517 - 1.1e^{-0.08(1-0.25)} \cdot 0.3515\right] \\
&= 4.4064
\end{aligned}
$$