Static Hedging of Asian Options under Lévy Models: The Comonotonicity Approach

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Abstract

In this paper we present a simple static super-hedging strategy for the payoff of an arithmetic Asian option in terms of a portfolio of European options. Moreover, it is shown that the obtained hedge is optimal in some sense. The strategy is based on stop-loss transforms and is applicable under general stock price models. We focus on some popular Lévy models. Numerical illustrations of the hegding performance are given for various Lévy models calibrated to market data of the S&P 500.

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1 Introduction

Pricing of (arithmetic) Asian options is even in the Black-Scholes world not straightforward. In general no explicit analytical expression for the average is available. So one has to use Monte Carlo simulation techniques to obtain numerical estimates of the price (see [9, 10, 23]), or one can follow a partial differential equation approach (cf. [18, 41]) (respectively a partial integro-differential equation approach in more general market models (cf. [42])). Both approaches are rather time consuming and the related hedging problem is even more difficult. Approximations of the distribution of the average that sometimes lead to closed-form expressions have also been studied (see e.g. [24, 39, 43]), but in general it is difficult to assess the approximation error and for hedging purposes this method is often not satisfying. For an approach based on Fast Fourier Transforms, see [7, 17].
An alternative route is to try to derive upper and lower bounds for the option price. This can nicely be done by the use of comonotonic theory as described in [19, 20, 38, 40]. We will follow this path and derive a static (super-)hedge for fixed-strike Asian call options based on a buy-and-hold strategy consisting of European call options maturing with and before the Asian option.
This is particularly useful since European call options are typically available on the market and quite liquidly traded. Moreover, only when the contract is struck, one has to take a position in these calls and no dynamic trading is needed.
Static hedging has several advantages over dynamic hedging. For instance, it is less sensitive to the assumption of zero transaction costs (both commissions and the cost of paying individuals to monitor the positions). Furthermore, dynamic hedging tends to fail when liquidity dries up or when the market makes large moves, but especially in these situations effective hedging is needed (see e.g. [12, 15, 16]).

As is illustrated in Section 4, the hedging error of our simple super-hedging strategy is very small if the option is in the money. For options at and out of the money this strategy can be quite conservative, but it is still much cheaper than the trivial super-hedge of the Asian option by a European option with identical strike and maturity (in case the dividend yield q is smaller than the continuously compound interest rate r). The procedure we develop is applicable for general stock price models. In this paper we focus on models where the asset price is described as the exponential of a general Lévy process. The special case of the NIG-Lévy process was considered in [1] and the case of the VG-process was covered in [2], where it was also observed that Asian option prices in these more realistic models differ significantly from the corresponding Black-Scholes prices. We will work out the theory in general, and in particular we will focus on the hedging problem.

In recent years it has been realized that the dynamics of stocks are much better described by a Lévy model than the classical Black-Scholes model. In a Lévy

model the Brownian motion is replaced by a more general Lévy process, taking into account the typical non-normality of asset returns. The stock price is modelled as the exponential of the Lévy process. Classical examples of Lévy processes used in this context are the VG-process, the NIG-process and the Meixner-process. For more examples and applications of Lévy processes in finance see [5, 13, 14, 21, 29, 36].
Lévy market models are, except in the Brownian and the unrealistic Poissonian case, incomplete. There are many candidates of equivalent martingale measures for risk-neutral valuation of derivative securities. Our approach is based on the risk-neutral densities of the distribution of the asset price and thus works for all equivalent martingale measures that lead to tractable numerical estimates of these density functions.

The paper is structured as follows. In Section 2 we describe how to obtain upper bounds for the price of an Asian option under a general market model using comonotonicity techniques. Next, we illustrate how to super-hedge Asian options using European call options in a buy-and-hold strategy. Section 3 describes the Lévy market model for asset prices and works out the theory in more detail for some popular examples such as the VG, the NIG and the Meixner case. Finally, in Section 4 we give numerical illustrations of the hedging strategy by calibrating all the models discussed in Section 3 to market data, namely a set of vanilla options on the S&P 500, and comparing the respective Monte Carlo prices, the comonotonic-upper-bound price (and the resulting static hedging strategy), with other (trivial) static super-hedges, including the well-known super-hedge by the European call with same strike and maturity, in case $q \leq r$.

2 A Static Hedging Strategy for Arithmetic Average Options

Throughout the text we will work under an arbitrage-free frictionless market model which consists of a riskless bond (bank account) and one financial risky asset, a stock or an index. The market dictates that there is a fixed interest rate $r \geq 0$, and that the bond price process behaves (deterministicly) as $B = \{B_t = \exp(rt), t \geq 0\}$. The stock price process follows a stochastic process and is denoted by $S = \{S_t, t \geq 0\}$. We assume that the stock pays a continuous compound dividend yield at a rate q per annum. We will always work with the natural filtration $\mathbb{F} = \mathbb{F}^S = \{\mathcal{F}_t, 0 \leq t \leq T\}$ of S. Later on, we will choose an exponential of a Lévy process for the stock price process, but first we develop the theory for a general model.
Suppose that in an arbitrary arbitrage-free incomplete market model we have selected an equivalent martingale measure Q, then the price of a European-style arithmetic average call option with strike price K, maturity T and n averaging

days $0 \leq t_1 < \ldots < t_n \leq T$ at time t is given by

$$\begin{aligned} AA_t &= \exp(-r(T-t))E_Q\left[\left(\frac{\sum_{k=1}^n S_{t_k}}{n} - K\right)^+ \Big| \mathcal{F}_t\right], \\ &= \frac{\exp(-r(T-t))}{n} E_Q\left[\left(\sum_{k=1}^n S_{t_k} - nK\right)^+ \Big| \mathcal{F}_t\right] \end{aligned}$$

where S_t is the asset price at time t, r is the risk-free interest rate and $(x-K)^+$ means $\max(x-K, 0)$.

The main difficulty in evaluating this expression is that in general the distribution of the average $\sum_{k=1}^n S_{t_k}/n$, which is a sum of dependent random variables, is not available. Here we focus on upper bounds based on a portfolio of European options. For that purpose, let us assume for simplicity that we are at time $t = 0$ and that the averaging has not yet started. First note, that for any $K_1, \ldots, K_n \geq 0$ with $K = \sum_{k=1}^n K_k$, we have a.s.

$$\left(\sum_{k=1}^n S_{t_k} - nK\right)^+ = \Big((S_{t_1} - nK_1) + \cdots + (S_{t_n} - nK_n)\Big)^+ \leq \sum_{k=1}^n (S_{t_k} - nK_k)^+ .$$

Hence

$$\begin{aligned} AA_0(K,T) &= \frac{\exp(-rT)}{n} E_Q\left[\left(\sum_{k=1}^n S_{t_k} - nK\right)^+ \Big| \mathcal{F}_0\right] \\ &\leq \frac{\exp(-rT)}{n} \sum_{k=1}^n E_Q\left[(S_{t_k} - nK_k)^+ \Big| \mathcal{F}_0\right] \\ &= \frac{\exp(-rT)}{n} \sum_{k=1}^n \exp(rt_k) EC_0(\kappa_k, t_k), \end{aligned} \tag{1}$$

where $EC_0(\kappa_k, t_k)$ denotes the price of a European call option at time 0 with strike $\kappa_k = nK_k$ and maturity t_k.

In terms of hedging this means that we have the following static super-hedging strategy: for each k, buy $\exp(-r(T-t_k))/n$ European call options at time $t = 0$ with strike κ_k and maturity t_k and hold these until their expiry. Then put their payoff on the bank account.

Since relation (1) holds for all combinations of $\kappa_k \geq 0$ that satisfy $\sum_{k=1}^n \kappa_k = nK$, we have a variety of portfolios of n European options whose payoff dominates the Asian option. For instance, the simplest choice is $\kappa_k = K$ ($k = 1, \ldots, n$). If $q \leq r$, we have $EC_0(K,t) \leq EC_0(K,T)$ for every $K \geq 0$ and $0 \leq t \leq T$, and thus this trivial choice shows that the Asian option price is dominated by the price of a European option with the same strike and maturity, i.e.

$$AA_0(K,T) \leq EC_0(K,T).$$

However, for our super-hedging purposes, we naturally look for that combination of κ_k's which minimizes the right-hand side of (1). As shown in Dhaene et al. [20], this optimal combination can be determined by using stop-loss transforms and the theory of comonotonic risks. In the following, we will briefly summarize these techniques and adapt them to our setting of general market models:
Let $F(x)$ be a distribution function of a non-negative random variable X, then (in accordance with actuarial practice) its stop-loss transform $\Phi_F(m)$ is defined by

$$\Phi_F(m) = \int_m^{+\infty} (x-m)\mathrm{d}F(x) = E[(X-m)^+], \quad m \geq 0.$$

A convex ordering of distribution functions $F(x)$ and $G(x)$ (or equivalently of the corresponding random variables) on the non-negative real line can be defined in the following way: $F(x)$ is said to precede $G(x)$ in convex order ($F \leq_{\mathrm{cx}} G$), if the corresponding means of the distribution functions (random variables) are equal and

$$\Phi_F(m) \leq \Phi_G(m) \text{ for all } m \geq 0.$$

If we write

$$A_n = \sum_{k=1}^{n} S_{t_k}$$

and $F^t_{A_n}(x) = \mathbb{P}_Q(A_n \leq x|\mathcal{F}_t)$ for the distribution function under Q of A_n given the information $\mathcal{F}_t$, then we have

$$AA_t = \frac{\exp(-r(T-t))}{n}\, \Phi_{F^t_{A_n}}(nK). \tag{2}$$

In this way the problem of pricing an arithmetic average option is transformed to calculating the stop-loss transform of a sum of dependent risks. Concretely, we will look at bounds for stop-loss transforms based on comonotonic risks: A positive random vector $(X_1,\ldots,X_n)$ with marginal distribution functions $F_1(x_1),\ldots,F_n(x_n)$ is called *comonotone*, if for the joint distribution function $F_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = \min\{F_1(x_1),\ldots,F_n(x_n)\}$ holds for every $x_1,\ldots,x_n \geq 0$. It immediately follows that the distribution of a comonotone random vector $(X_1,\ldots,X_n)$ with given marginal distributions $F_1(x_1),\ldots,F_n(x_n)$ is uniquely determined.
In [38], it was shown that an upper bound for the stop-loss transform of the sum of arbitrary dependent positive random variables $\sum_{k=1}^n X_k$ with marginal distributions $F_1(x_1),\ldots,F_n(x_n)$ is given by the stop-loss transform of the sum $S^c = \sum_{k=1}^n Y_k$, where $(Y_1,\ldots,Y_k)$ is the comonotone random vector with marginal distributions $F_1(x_1),\ldots,F_n(x_n)$, i.e.

$$\sum_{k=1}^{n} X_k \leq_{\mathrm{cx}} \sum_{k=1}^{n} Y_k.$$

Let $F_{S^c}(x)$ denote the distribution function of $\sum_{k=1}^n Y_k$, then we have the following relation for its inverse

$$F_{S^c}^{-1}(x) = \sum_{k=1}^{n} F_{X_k}^{-1}(x), \quad x \geq 0. \tag{3}$$

From Theorem 6 in [38] it follows that the stop-loss transform of a sum of comonotonic random variables can be obtained as a sum of the stop-loss transforms of the marginals evaluated at specified points, namely

$$\Phi_{F_{S^c}}(m) = \sum_{k=1}^{n} \Phi_{F_{X_k}}\left(F_{X_k}^{-1}(F_{S^c}(m))\right), \quad m \geq 0, \tag{4}$$

given that the marginal distribution functions involved are strictly increasing (which will always be the case in our applications). At the same time, we have

$$\Phi_{F_{S^c}}(m) = E\Big(\Big(\sum_{k=1}^{n} Y_k - m\Big)^+\Big) \leq \sum_{k=1}^{n} E\left((Y_k - m_k)^+\right) = \sum_{k=1}^{n} \Phi_{F_{X_k}}(m_k) \tag{5}$$

whenever $\sum_{k=1}^n m_k = m$. Thus the stop-loss transform of the comonotonic sum given by (4) at the same time represents the lowest possible bound in terms of a sum of stop-loss transforms of the marginal distributions.

We will now apply this result to our setting of an arithmetic Asian option. Let $F(x_k;t_k)$ $(k = 1,\dots,n)$ denote the conditional distribution of S_{t_k} under the risk-neutral measure Q (given the information available at time $t = 0$), i.e. for $x_k, t_k > 0$,

$$F(x_k;t_k) \quad = \quad \mathbb{P}_Q\left(S_{t_k} \leq x_k | \mathcal{F}_0\right). \tag{6}$$

Combining (1), (2), (4) and (5), we thus have found the optimal combination of strike prices κ_k, namely

$$\kappa_k = F^{-1}\left(F_{S^c}(n\,K);t_k\right), \quad k = 1,\dots,n. \tag{7}$$

In that way, we have obtained the optimal static super-hedge in terms of European call options with maturity dates equal to the averaging dates.

For the practical determination of the strike prices κ_k, the distribution function of the comonotone sum $F_{S^c}(x)$ as given by (3) has to be calculated and evaluated at nK (note that the involved marginal distribution functions are strictly increasing and continuous). In case the risk-neutral density (or an approximation of it) is available, this can be done numerically in a straight-forward way (cf. Section 4). The κ_k's are then obtained by evaluating the inverse distribution function of $F(x;t_k)$.

3 The Lévy Market Model

Suppose $\phi(u)$ is the characteristic function of a distribution. If for every positive integer n, $\phi(u)$ is also the nth power of a characteristic function, we say that the distribution is infinitely divisible.
One can define for every such infinitely divisible distribution a stochastic process, $X = \{X_t, t \geq 0\}$, called Lévy process, which starts at zero, has independent and stationary increments and such that the distribution of an increment over $[s, s+t]$, $s, t \geq 0$, i.e. $X_{t+s} - X_s$, has $(\phi(u))^t$ as its characteristic function.
Every Lévy process has a càdlàg modification which is itself a Lévy process. We always work with this càdlàg version of the process. So sample paths of a Lévy process are a.e. continuous from the right and have limits from the left. The cumulant characteristic function $\psi(u) = \log \phi(u)$ is often called the *characteristic exponent* (see e.g. [8]).
We assume our market to consist of one riskless asset (the bond) with price process given by $B_t = \exp(rt)$ and one risky asset (the stock or index). The risk-neutral model for the risky asset is given by

$$S_t = S_0 \frac{\exp((r-q)t)}{E[\exp(X_t)]} \exp(X_t).$$

The factor $\exp((r-q)t)/E[\exp(X_t)]$ puts us immediately in a risk-neutral setting by a mean correcting argument. Note that the argument underlying the above choice of a risk-neutral measure is in line with the classical risk-neutrally mean-correcting technique used in the Black-Scholes setting. We would like to stress, however, that our proposed hedging strategies are not restricted to this particular choice of a risk-neutral density.

Note that in this case we have for (6)

$$\begin{aligned} F(x_k; t_k) &= \mathbb{P}_Q \left(S_{t_k} \leq x_k | \mathcal{F}_0 \right) && (8) \\ &= \mathbb{P}_Q \left(S_0 \frac{\exp((r-q)t)}{E[\exp(X_t)]} \exp(X_{t_k}) \leq x_k | \mathcal{F}_0 \right) && (9) \\ &= \mathbb{P}_Q \left(X_{t_k} \leq \log(x_k/S_0) + \psi(-\mathrm{i}) - (r-q)t \middle| \mathcal{F}_0 \right) && (10) \end{aligned}$$

In the next section, we describe three popular Lévy processes, which are often used in the modelling of financial assets: the VG process, the NIG process and the Meixner process.
To obtain the price $EC(K, T)$ of a European call option with strike K and time to maturity T under these models, one can use the Carr and Madan formula [11], which is formulated in terms of the characteristic function of the underlying Lévy process: Let α be a positive constant such that the αth moment of the stock price exists (typically a value of $\alpha = 0.75$ will do fine). Then

$$EC(K, T) = \frac{\exp(-\alpha \log(K))}{\pi} \int_0^{+\infty} \exp(-\mathrm{i}v \log(K)) \varrho(v) \mathrm{d}v, \qquad (11)$$

where

$$\begin{aligned}\varrho(v) &= \frac{\exp(-rT)E[\exp(\mathrm{i}(v-(\alpha+1)\mathrm{i})\log(S_T))]}{\alpha^2+\alpha-v^2+\mathrm{i}(2\alpha+1)v} \qquad (12)\\ &= \frac{\exp(-rT)\phi(v-(\alpha+1)\mathrm{i})}{\alpha^2+\alpha-v^2+\mathrm{i}(2\alpha+1)v}. \qquad (13)\end{aligned}$$

The Fast Fourier Transform can be used to invert the generalized Fourier transform of the call price. Using the above formula one can typically calculate the complete option surface over all strikes and maturities in a fraction of a second.

3.1 Concrete Examples

3.1.1 The Variance Gamma Process

The VG(C, G, M) law has a characteristic function of the form

$$\phi_{VG}(u; C, G, M) = \left(\frac{GM}{GM + (M-G)\mathrm{i}u + u^2}\right)^C,$$

and its density function is given by

$$\begin{aligned} f_{\mathrm{VG}}(x; C, G, M)(x) &= \frac{(G\,M)^C}{\sqrt{\pi}\,\Gamma(C)} \exp\left(\frac{(G-M)x}{2}\right) \\ &\quad\times \left(\frac{|x|}{G+M}\right)^{C-1/2} K_{C-1/2}\big((G+M)\,|x|/2\big) \qquad (14)\end{aligned}$$

where $\mathrm{K}_\nu(x)$ denotes the modified Bessel function of the third kind with index ν, $\Gamma(x)$ denotes the gamma function and $C, G, M > 0$. This distribution is infinitely divisible and has the following convolution property: $\phi_{VG}(u; C, G, M) = (\phi_{VG}(u; C/n, G, M))^n$. Thus one can define the VG-process $X^{(VG)} = \{X_t^{(VG)}, t \geq 0\}$ as the process which starts at zero, has independent and stationary increments and where the increment $X_{s+t}^{(VG)} - X_s^{(VG)}$ over the time interval $[s, t+s]$ follows a VG(Ct, G, M) law.
Note that sometimes another parameterization of the VG distribution is used (see e.g. [36]).
The class of Variance Gamma distributions as a model for stock returns was introduced by [27] in the late 1980s (where the symmetric case $G = M$ was considered, see also [26] and [28]). In [25], the general case with skewness is treated.

3.1.2 The Normal Inverse Gaussian Process

The Normal Inverse Gaussian (NIG) distribution with parameters $\alpha > 0$, $|\beta| < \alpha$ and $\delta > 0$, has a characteristic function given by:

$$\phi_{NIG}(u; \alpha, \beta, \delta) = \exp\left(-\delta\left(\sqrt{\alpha^2 - (\beta + \mathrm{i}u)^2} - \sqrt{\alpha^2 - \beta^2}\right)\right).$$

Again, one can clearly see that this is an infinitely divisible characteristic function with $\phi_{NIG}(u;\alpha,\beta,\delta) = (\phi_{NIG}(u;\alpha,\beta,\delta/n))^n$. Hence we can define the NIG-process $X^{(NIG)} = \{X_t^{(NIG)}, t \geq 0\}$, with $X_0^{(NIG)} = 0$, stationary and independent NIG distributed increments: To be precise, $X_t^{(NIG)}$ has a NIG$(\alpha,\beta,t\delta)$ law. The density of the NIG(α,β,δ) distribution is given by

$$f_{NIG}(x;\alpha,\beta,\delta) = \frac{\alpha\delta}{\pi}\exp\left(\delta\sqrt{\alpha^2-\beta^2}+\beta x\right)\frac{K_1(\alpha\sqrt{\delta^2+x^2})}{\sqrt{\delta^2+x^2}}.$$

The NIG distribution was introduced by [3]. See also [4], [30] and [31].

3.1.3 The Meixner Process

The density of the Meixner distribution is given by

$$f_{Meixner}(x;\alpha,\beta,\delta) = \frac{(2\cos(\beta/2))^{2\delta}}{2\alpha\pi\Gamma(2d)}\exp\left(\frac{bx}{a}\right)\left|\Gamma\left(\delta+\frac{\mathrm{i}x}{\alpha}\right)\right|^2,$$

where $\alpha > 0, -\pi < \beta < \pi, \delta > 0$.
The characteristic function of the Meixner(α,β,δ) distribution is given by

$$\phi_{Meixner}(u;\alpha,\beta,\delta) = \left(\frac{\cos(\beta/2)}{\cosh\frac{\alpha u-\mathrm{i}\beta}{2}}\right)^{2\delta}.$$

The Meixner(α,β,δ) distribution is infinitely divisible: $\phi_{Meixner}(u;\alpha,\beta,\delta) = (\phi_{Meixner}(u;\alpha,\beta,\delta/n))^n$. It thus generates a Lévy process which we call the Meixner process. More precisely, a Meixner process $X^{(Meixner)} = \{X_t^{(Meixner)}, t \geq 0\}$ is a stochastic process which starts at zero, i.e. $X_0^{(Meixner)} = 0$, has independent and stationary increments, and where the distribution of $X_t^{(Meixner)}$ is given by the Meixner distribution Meixner$(\alpha,\beta,\delta t)$.
The Meixner process was introduced in [32] (see also [33]) and later on it was suggested to serve for fitting stock returns in [22]. This application in finance was worked out in [34] and [35].

4 Numerical Results

We will now illustrate the performance of the static hedge-portfolio for the market models discussed in Section 3.1 applied to a liquid market. Concretely, we will calibrate our model parameters to the set of vanilla options on the S&P 500 as given in [36, Appendix C]. The yearly risk-free interest rate and the dividend rate are given by $r = 0.019$ and $q = 0.012$, respectively. The result of the calibration in the least squared sense, i.e. with the minimal value of

$$\text{lse} = \sum_{options}(\text{Market price} - \text{Model price})^2,$$

Model	**Parameters**	
VG		
C	G	M
1.3574	5.8704	14.2699
NIG		
α	β	δ
6.1882	-3.8941	0.1622
Meixner		
α	β	δ
0.3977	-1.4940	0.3462

Table 1: Lévy models (mean correcting): parameter estimation

is given in Table 1.
We investigate an arithmetic Asian call option with a maturity of 1 year and averaging every month (i.e. 12 averaging days). In order to set up our hedge portfolio, we thus have to determine the inverse distribution function of the asset price at these 12 days (cf. (6)). This is done by discretizing the real line in an appropriate range and numerically building up the distribution function from the density function. The inverse is then found by a bisection method from the corresponding table and linear interpolation between grid points is employed. It turns out that using 40000 points in the grid is sufficient (in the sense that a further increase does not change the significant digits of the results). Next, the inverse of the distribution of the comonotone sum is built up according to (3) and then itself inverted in the above way. Finally, the strike prices κ_k of the European options are obtained by evaluating the inverse distribution functions of the marginals according to (7). For the models discussed in Section 3.1, this numerical procedure to obtain the strike prices for our hedging strategy is both accurate and very quick (it takes less than a minute on a normal PC to determine the entire hedge portfolio).

In Tables 2 and 3 the strike prices as a percentage of the spot price are listed for the above example and the various models calibrated to the S&P 500 (all numbers are rounded to their last digit). Note that the optimal strike prices hardly differ among the various models considered.

The price of the hedging strategy is then easily determined using the European call option pricing formula (11) of Carr and Madan and (1). Tables 4-6 compare the Monte-Carlo simulated price of the Asian option AA_{MC} and the comonotonic superhedge price AA_c, with the prices of two trivial super-hedging strategies, namely the trivial super-hedge using the European option price EC with identical strike and maturity (note $q \leq r$) and the super-hedge (1) with all $\kappa_i = K$ with price AA_{tr}.
For the Monte-Carlo price, we used 1000000 simulated paths. The VG process was simulated as a difference of 2 Gamma processes (cf. [36, Section 8.4.2]),

NIG paths were obtained as described in [36, Section 8.4.5] and Meixner paths were obtained by a compound poisson approximation as described in [36, Section 8.2.1].

From Tables 4-6 we observe that the more in the money the Asian option is, the less is the difference between the option price and the comonotonic hedge. For an option with moneyness of 80% the difference is typically around 1.5%, whereas the classical hedge with the European call leads to a difference of almost 10%. For options out of the money, the difference increases, but is then substantially smaller than the differences for the other two trivial hedges. In view of the easy and cheap way in which this hedge can be implemented in practice, this comonotonic approach seems to be competitive also in these cases.

5 Conclusion

Pricing of exotic derivatives is in general on rather weak foundations. As was recently realized (see e.g. [37]), calibration of a variety of market models may lead to widely differing prices of exotic options, which underlines the fact that obtaining concrete super-hedging strategies is of utmost importance. We have shown that staticly hedging an Asian option in terms of a portfolio of European options is a simple and quick alternative to existing tools. Moreover, opposed to most of the existing techniques, this approach is applicable in general market models whenever the risk-neutral density of the asset price distribution or an approximation of it is available. Since the proposed hedging strategy is static, it is much less sensitive to the assumption of zero transaction costs and to the hedging performance in the presence of large market movements; no dynamic rebalancing is required. These advantages may sometimes compensate the gap of the hedging price and the option price even for OTM Asian options.

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$100K/S_0$	t_k	κ_k (VG model)	κ_k (NIG model)	κ_k (Meixner model)
80	0.083	92.82	94.87	95.14
	0.167	89.23	90.66	90.65
	0.250	86.39	87.23	87.03
	0.333	83.59	84.34	84.07
	0.417	81.42	81.84	81.57
	0.500	79.54	79.62	79.41
	0.583	77.87	77.64	77.51
	0.667	76.37	75.83	75.80
	0.750	75.00	74.18	74.24
	0.833	73.74	72.65	72.82
	0.917	72.56	71.23	71.50
	1.000	71.46	69.90	70.27
90	0.083	98.36	98.23	98.50
	0.167	96.56	96.40	96.71
	0.250	94.86	94.67	94.90
	0.333	92.88	93.06	93.19
	0.417	91.30	91.56	91.59
	0.500	89.91	90.17	90.12
	0.583	88.64	88.86	88.75
	0.667	87.49	87.62	87.48
	0.750	86.43	86.46	86.29
	0.833	85.43	85.36	85.18
	0.917	84.50	84.31	84.13
	1.000	83.62	83.31	83.14
100	0.083	100.84	100.33	100.38
	0.167	101.33	100.48	100.57
	0.250	101.49	100.51	100.63
	0.333	101.11	100.47	100.59
	0.417	100.75	100.38	100.48
	0.500	100.36	100.25	100.32
	0.583	99.96	100.09	100.12
	0.667	99.57	99.91	99.90
	0.750	99.19	99.72	99.65
	0.833	98.82	99.51	99.39
	0.917	98.46	99.29	99.13
	1.000	98.11	99.06	98.85

Table 2: Strike prices for the hedge portfolio (S&P 500) (Part 1)

$100K/S_0$	t_k	κ_k (VG model)	κ_k (NIG model)	κ_k (Meixner model)
110	0.083	101.87	102.35	102.24
	0.167	103.61	104.21	104.09
	0.250	105.24	105.83	105.72
	0.333	106.93	107.28	107.19
	0.417	108.42	108.62	108.56
	0.500	109.81	109.86	109.84
	0.583	111.12	111.03	111.04
	0.667	112.36	112.14	112.18
	0.750	113.53	113.20	113.27
	0.833	114.65	114.20	114.31
	0.917	115.72	115.17	115.30
	1.000	116.74	116.10	116.26
120	0.083	106.63	106.05	105.86
	0.167	109.81	109.68	109.55
	0.250	112.52	112.61	112.54
	0.333	115.20	115.18	115.16
	0.417	117.54	117.53	117.54
	0.500	119.71	119.73	119.75
	0.583	121.76	121.80	121.84
	0.667	123.72	123.78	123.84
	0.750	125.60	125.69	125.75
	0.833	127.42	127.54	127.61
	0.917	129.19	129.33	129.40
	1.000	130.91	131.07	131.15

Table 3: Strike prices for the hedge portfolio (S&P 500) (Part 2)

$100K/S_0$	AA_{MC}	AA_c	AA_{tr}	EC
80	20.5233	20.7895	20.9331	22.0739
90	11.7384	12.1649	12.3462	14.2015
100	4.5979	5.0555	5.0764	7.7732
110	0.9585	1.2261	1.5090	3.3712
120	0.2108	0.3364	0.4824	1.2554

Table 4: VG option prices as percentage of the spot (S&P 500)

$100K/S_0$	AA_{MC}	AA_c	AA_{tr}	EC
80	20.6067	20.9335	21.0906	22.3345
90	11.7500	12.2184	12.3885	14.3309
100	4.4899	5.0184	5.0223	7.7433
110	0.9208	1.2477	1.5039	3.3441
120	0.1865	0.3149	0.4660	1.2381

Table 5: NIG option prices as percentage of the spot (S&P 500)

$100K/S_0$	AA_{MC}	AA_c	AA_{tr}	EC
80	20.7128	20.8870	21.0459	22.2530
90	11.8590	12.2050	12.3861	14.3029
100	4.5133	5.0147	5.0204	7.7499
110	0.8768	1.2471	1.5085	3.3476
120	0.1961	0.3382	0.4862	1.2601

Table 6: Meixner option prices as percentage of the spot (S&P 500)