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Volume Four

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Understanding Credit Derivatives

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FOREWORD

The premium paid/received on a CDS represents the market's view of the reference entity's credit risk over the duration of the CDS transaction. Obviously such views are also reflected in the yield spreads of the reference entity's debt. This means that CDS pricing is highly linked to bond spreads. In fact, arbitrage trading between the CDS and bond markets drives pricing in the two markets to a common range.

The liquid market quotes of CDS premiums are mainly driven by the arbitrage relationship with bond spreads, rather than priced from a model. Pricing off-market CDS is quite a different matter. These are outstanding transactions some time after inception, which do not have any observable market quotes. Their pricing therefore needs to be generated from a quantitative model.

In the first chapter of this volume, we show the arbitrage relationship between CDS and bond spreads. We then present the basics of the quantitative CDS pricing model, including the equations and derivation of default probabilities.

In the second chapter of this volume, we examine the mark-to-market ("MTM") calculations for seasoned CDS, highlighting the use of implicit default probabilities. We also review the MTM differences between CDS and asset swaps. Finally, we discuss the mechanisms used to "unwind" (cancel or hedge) existing CDS transactions.

CHAPTER 1: CREDIT DEFAULT SWAP PRICING

1. ASSET SWAP SPREAD AS A PRICING BENCHMARK

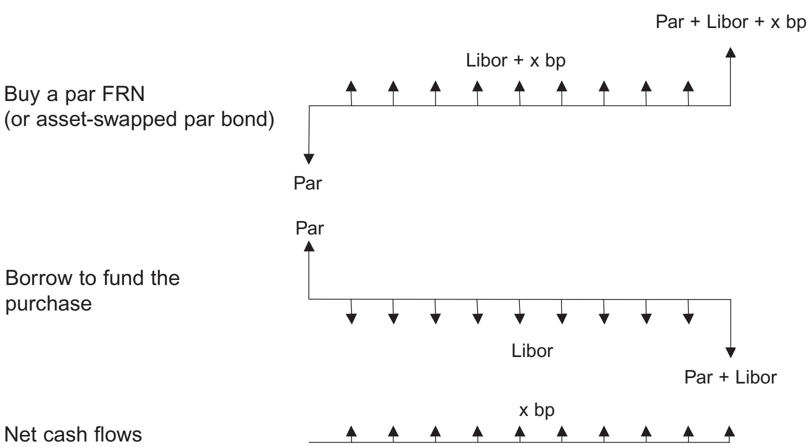
CDS pricing makes reference to bond market pricing, specifically yield spreads

A credit default swap transfers defined credit risk between counterparties, so its pricing, or premium, represents the market's view of the reference entity's credit risk over the duration of the CDS transaction. Obviously such views are already reflected in the yield spreads of the reference entity's debt. This means that CDS pricing is highly linked to bond spreads, and that arbitrage trading drives pricing in the CDS and bond markets to a common range.

But there are significant structural differences between a CDS and a bond. A CDS is unfunded, meaning that, unlike a bond, there is no initial outlay of the notional amount, nor any principal repayment at maturity. There is only a stream of periodic premium payments until the earlier of a credit event or maturity, and if a credit event occurs, a settlement payment of (1 - recovery rate) * notional amount.

CDS can be replicated by a financed purchase of a par-priced FRN

Chart 1: Replicating a CDS (sell protection, assuming no credit event)



Source: BNP Paribas

To replicate a CDS (selling protection, for example) using bond market instruments, we can buy a par floating rate note or an asset-swapped fixed-rate par bond[1] and borrow money to fund the purchase. The resulting risk profile of this package is similar to that of a CDS: long credit risk with interest rate risk hedged (there are, however, certain differences, which we illustrate at the end of this section). The resulting cash flow profile is also similar (see Chart 1):

[1] Throughout this paper, we will refer to the "cash market". This refers to debt, either FRNs or fixed-rate bonds asset-swapped into floaters.

- Borrowing is necessary here so that the bond's principal cash flows on the purchase date and at maturity are offset by the borrowing and repayment cash flows, leaving the net cash flows matching those of an unfunded CDS.
- For fixed-rate bonds, an asset swap is necessary so that interest rate risk is hedged and credit risk is segregated.
- Borrowing is done through the repo market to achieve attractive funding levels. If we assume Libor to be the implicit long-term repo rate for the replication, the residual cash flows from the package will be only a stream of periodic payments representing the spread over Libor (we will call this the "cash spread". We discuss the case where the funding costs are different from Libor in the following section).
- Here we take the spread over Libor to represent the credit risk. Although this is not necessarily true in theory (credit spread should be the spread over government bonds), in practice Libor is often used as the effective risk-free rate for derivative pricing.
- Most importantly, the bond used for replication should trade at, or close to, par, as the CDS notional is par. Otherwise the CDS premium will tend to diverge from the spread of the package (further discussions in Section 3 of this chapter).

CDS premium is usually close to the spread over Libor of a par bond

Since the above package has similar risk and cash flow profiles to those of a CDS, the premium should be close to the spread over Libor of a par bond.

One notable exception is that bonds issued by highly-rated borrowers may trade at sub-Libor levels. But in the CDS market their premiums have a floor of zero (cannot be negative).

There are, however, differences between the replication and a CDS

The replication demonstrates a link between CDS premium and cash spread. However, as we mentioned earlier, the risk and cash flow profiles of the replication package and a CDS, while similar, are not identical. The following structural differences can result in a divergence of CDS premium and cash spread[2]:

- The ongoing premium of a CDS is contingent on the non-occurrence of a credit event, whereas in the asset swap package, the interest rate swap component is not credit-contingent. The asset swap buyer will continue to receive the floating-rate coupon and pay fixed, even if the asset defaults. Obviously, he will have lost the fixed coupon income from the now-defaulted asset to fund his fixed-rate payment obligation, and will probably need to unwind this swap, subjecting him to a mark-to-market gain or loss.
- A defaulted bond does not pay the coupon accruals from the last coupon date to the default date, whereas a CDS protection buyer pays the accrued premium.
- The "risky" (contingent) nature of CDS' cash flows means that, for credits trading below par, CDS premiums tend to be higher than asset swap spreads, which is called a positive basis (see Section 3 for details).

[2] Apart from these structural differences, supply and demand relationships in the CDS and cash markets, investors' liquidity considerations and hedging activities can also result in a positive or negative basis (basis = CDS premium – asset swap spread).

- CDS also have certain unique features, such as the cheapest-to-deliver option, which push CDS premiums higher (recent changes to standard documentation, such as the introduction of Modified Restructuring[3] and Modified Modified Restructuring , reduce the value of this option). On the other hand, a protection buyer has counterparty credit risk to the protection seller. Specifically, he has the risk that after the reference entity defaults, the protection seller also defaults on the CDS contract, i.e., the protection buyer is subject to the default correlation between the seller and the reference entity. This counterparty risk reduces the CDS premium that the protection buyer is willing to pay.

These structural differences, together with other market factors, may result in a divergence between CDS premium and cash spread (in the following two sections we examine some of the most important forces that drive such a divergence). Nevertheless, the replication that we have discussed here is still a starting point, and asset swap spreads serve as pricing benchmarks for CDS.

3 See *Understanding Credit Derivatives Volume 2: CDS Basics*

2. FUNDING COST DIFFERENCES

The illustration in the previous section assumed that all market participants funded themselves at one single level in the repo market, which was Libor. Therefore the replication would only result in a CDS premium of x bp, i.e., there was a one-to-one relationship between cash spread and CDS premium. In reality, however, different market players have different funding levels. Top quality borrowers can fund at sub-Libor rates, while others may have to fund at Libor plus[4]. These cost differentials result in a breakdown of the one-to-one relationship, and introduce a theoretical range around the cash spread within which CDS may trade. Consider the following example:

Differences in funding costs introduce a range around cash spread within which the CDS may trade...

- One AAA rated institution funds at L-10bp;
- One single-A rated institution funds at L+10bp.

Accordingly, the CDS premium from the replication would be:

- x+10bp for the AAA institution; and
- x-10bp for the single-A institution[5].

Assuming that these are the only players in the CDS market, the above levels would be the boundaries of the CDS' theoretical trading range, i.e., the CDS could trade as low as x-10bp or as high as x+10bp. Where the CDS actually trades will depend on the supply and demand for protection from these players, which are in turn influenced by certain arbitrage activities between them as described below.

Suppose, as a start, the CDS trades at x bp, the same as the cash spread. This does not prevent the following arbitrage activities:

...and arbitrage activities help keep the CDS trading within that range

- The AAA institution buys the bond and buys protection. It borrows at L-10bp, buys the bond yielding L+xbp, and pays x bp for protection, thereby locking in a 10bp profit with zero net exposure to the bond[6]. This arbitrage comes from its sub-Libor funding cost advantage.
- The single-A institution sells CDS protection outright. It earns x bp, whereas if it borrows to buy the bond, it earns only x-10bp. It avoids the Libor-plus funding cost disadvantage, and effectively achieves funding at Libor through the CDS position.

So the AAA institution buys protection from the single-A one. The supply-demand equilibrium between them determines the actual level of CDS premium. In any case, these arbitrage activities keep the CDS premium within a certain range of cash spread, linking the two markets.

As we can see, these arbitrage activities are driven by differences in funding costs, where higher-rated entities are protection buyers and lower-rated entities protection sellers. Nevertheless, we cannot generalise that all protection buyers in the CDS market have higher credit quality than protection sellers, because people participate in the market for reasons other than funding cost arbitrage. For example, banks, no matter whether rated AAA or A, can be net buyers of protection for purposes of managing their regulatory or economic capital.

In funding cost arbitrage, institutions with low funding costs are protection buyers, and those with high funding costs protection sellers

4 Repo rates are affected not only by the borrowers' credit quality, but also by the rating and liquidity of the bond, the default correlation between the borrower and the bond issuer, and general market dynamics. Currently, the funding cost differential due to counterparty rating differences is not large.

5 Assuming x > 10bp, as CDS premium cannot be negative.

6 It incurs credit exposure to the CDS counterparty though. This counterparty credit risk, however, can be mitigated by certain collateral facilities between counterparties such as the ISDA Credit Support Annex ("CSA").

3. DISCOUNTED BONDS – ADDITIONAL CAPITAL AT RISK

When bonds trade below par, CDS protection sellers have additional capital at risk, and therefore require higher premiums

We have used par bonds to illustrate the relationship between CDS premium and cash spread. When bonds trade significantly away from par, this relationship breaks down because of the structural differences between bonds and CDS. For a bond investment, the capital at risk for the buyer is the market value of the bond less any recovery value, whereas the exposure for a CDS protection seller is par less recoveries. Therefore, when bonds trade below par the CDS protection seller has more capital at risk than the bond buyer – this is roughly equal to the discount of the bond.

Obviously the protection seller will require additional compensation to cover this risk. This results in a higher CDS premium than cash spread (positive basis).

For distressed credits, the conventional CDS quotation method often fails

When a credit becomes distressed and its bonds trade at a steep discount, the capital at risk in CDS transactions will be so large that even high running premiums may not attract protection sellers. In addition, there is more risk that they will not receive the high running premiums for long. This in turn has an accelerating effect on the premium, raising it even higher and well above the asset swap spread (we discuss the role of implied default probabilities in CDS pricing in the next section). This is why CDS premiums usually rise faster than asset swap spreads when bond prices fall below par. In the extreme case where a bond trades close to its expected recovery value, the implicit default probability is almost 100%, i.e., imminent. Since the protection seller can only expect to receive the premium for a short period of time, the theoretical premium should rise substantially. The conventional CDS quotation method breaks down.

From the protection buyers' perspective, the conventional quotation method for distressed credits subjects them to the risk that, if a credit event does not occur soon, they will be locked into high premiums possibly for longer than is initially expected. Because of such risk aversions from both counterparties, when premiums rise towards 1000bp, few trades can be done using the conventional quotation method. An alternative method is needed.

Distressed credits are often quoted on a "points-upfront" basis...

This alternative method is "points upfront", which takes either of these two forms:

- one single payment only quoted in bond points, paid at the initiation of the CDS contract, for protection until maturity; or
- one upfront payment quoted in bond points plus a running premium paid until the earlier of a credit event or maturity. Obviously in this case the running premium is much lower than it would be under the conventional quotation method.

Points-upfront CDS are usually traded in maturities of up to one year, although longer maturities are also a possibility. This is because investors are most concerned with the prospects of imminent default.

The amount of the upfront payment, or the combined present value of an upfront payment plus running premiums, will be the same as the expected present value of the otherwise high running premiums. In any case, they should all be equal to the expected present value of the contingent protection payment, which is 1 – Recovery[7]. However, points-upfront CDS addresses counterparties' risk aversion in two ways:

[7] See the next section for generic CDS pricing equations.

...as it reduces the uncertainty about the premium cash flows

- It removes or reduces the uncertainty about the premium cash flows[8].
- It also reduces the CDS transaction's profit-and-loss sensitivity to future spread movements, as the premium cash flow is front-loaded and its duration reduced.

Points-upfront is therefore a more acceptable solution to both counterparties. Consequently, distressed credits trade on a points-upfront basis much more often than on a running premium basis. That said, the upfront payment does mean that the protection buyer has significant counterparty credit exposure to the seller. Again, this can be mitigated by certain collateral arrangements such as ISDA Credit Support Annex.

Obviously, these two methods can give different payoffs, depending on the timing of the credit event. For example, a protection buyer will be better off using the conventional running premium method if a credit event happens sooner than expected; on the other hand, he will be better off using the points-upfront method if the credit event happens later than expected or if the credit survives to maturity. Therefore, a participant's view on the likelihood and timing of a credit event affects his choice of the quotation method. Furthermore, his choice is constrained by his counterparty's willingness to transact on the same quotation basis.

CDS' single upfront payment is larger than the discount on the bond

Lastly, it is important to note that the single upfront payment amount is always larger than the discount on the bond. We prove this by showing an example that the reverse is not possible. Suppose a bond trades at $90 and the CDS at $10 upfront. Buying the bond and protection costs $100 and the investor will get the $100 back no matter whether default occurs (if default occurs, he can deliver the bond to the protection seller for par). In addition, he will also get the bond's coupons until default. Further assuming that the coupon is higher than his funding cost (which is normally the case), he would be receiving a stream of excess spreads for free, should the CDS' upfront payment equal the discount on the bond. This would represent a risk-free arbitrage. Therefore, the sum of the bond price and the CDS' single upfront payment must be above par, meaning that the upfront payment is more than the discount on the bond.

[8] In contrast, if a protection seller using the conventional running premium method wants to hedge his contingent premium income, he will need to buy a strip of CDS whose maturities correspond to the payment dates of the premiums being hedged, and whose expected payoffs upon a credit event equal the loss of the premiums (we discuss hedging with a strip of CDS in more detail in Section 3 of the next chapter). Because of the difference in cash flow uncertainty, combining an upfront CDS with a running-premium CDS on the same reference entity (such as selling protection on a running basis and buying protection upfront) results in zero net position in credit event payout, but leaves the running premium leg unhedged. The net effect is a short protection position in a strip of CDS.

4. QUANTITATIVE PRICING MODELS

As we have illustrated, the premium of a new CDS referencing a non-distressed entity will be determined mostly by the arbitrage relationship with the cash spreads of that entity[9]. However, once the CDS is seasoned and spreads have moved, quantitative models are used for pricing.

Intuitively, CDS premiums should be determined by the following factors:

- the default probability of the reference entity;
- the expected recovery rate of the deliverable obligations;
- the maturity of the swap;
- the default probability of the protection seller, as well as the default correlation between the protection seller and the reference entity.

Quantitative models are used in pricing off-market CDS

The liquid market quotes of CDS premiums are mainly driven by the arbitrage relationship with bonds and supply-demand factors, rather than priced from a quantitative model. In fact, these quotes are themselves inputs into pricing models, as explained later. On the other hand, quantitative models are necessary for pricing off-market CDS, such as outstanding transactions some time after inception, which have no liquid market quotes.

A fundamental concept for CDS pricing is that a CDS transaction has zero net present value at inception for both parties. This means that the present value ("PV") of the premium leg and the PV of the default leg must be equal, i.e., the premium is set at a level that equates the PVs of the two legs[10](we discuss the PV calculation methods below).

The premium of an off-market CDS re-establishes the equation between the PVs of the premium leg and the default leg

After inception, however, as expectations change, the PVs of the two legs will be different. If the on-market CDS quotes for a reference entity have tightened, protection sellers will have a positive mark-to-market ("MTM"), and vice versa when premiums widen. In either case, the market price of this off-market (seasoned) CDS is different from its contract premium. The market price, or the intrinsic value of protection, is the level which equates the PVs of the two swap legs again. A quantitative model is used to derive this intrinsic value.

[9] The CDS market, having gained significant liquidity over the past few years, has become a mainstream marketplace for pricing credit risk. In other words, it is not always cash spreads driving CDS premiums, but often the other way round. The arbitrage relationship represents an interaction between the two markets.

[10] They are equal in absolute value but with opposite signs, so that the sum of the two PVs equals zero.

The default probability model

The question now is how to calculate the PVs. Both legs are uncertain cash flows: premiums are paid up to the earlier of a credit event or maturity, and the payment of 1 - Recovery only happens upon occurrence of a credit event before the maturity of the swap. This uncertainty means that the PVs have to be calculated with credit event probabilities, and these probabilities are paramount to CDS pricing. This is why a CDS pricing model is also referred to as a *default probability model*.

PVs are calculated with default probabilities...

This is where the on-market CDS quotes for the reference entity come into play, as default probabilities are inferred from these quotes. Inferred probabilities rather than rating agencies' historical default rates are used, so as to ensure that the off-market pricing is in line with on-market quotes. In other words, the pricing model is calibrated to the market.

...which are inferred from on-market CDS premiums

CDS premiums represent both the probability of default and the loss given default (i.e., 1 - Recovery)[11]. Spread, default probability and recovery rate form a credit triangle, represented by the following idealised equation[12]:

A recovery rate assumption needs to be made

$$\textit{Hazard Rate} \approx \textit{Premium} \,/\, (1 - \textit{Recovery})$$

In the credit triangle, knowing any two of the elements leads to a solution of the third. However, the CDS premium is the only item that is observable from the market. In order to solve for hazard rates (and therefore default probabilities), a recovery rate assumption must be made.

For CDS referencing senior unsecured obligations, a recovery rate of 40% is often used, which approximates the historical long-term average recovery rate for debt of this level[13] (for background information on recovery rates, see our report "*Corporate Default, Recovery and Rating Transition Statistics for 2003", March 2004*). Currently, some dealers are making an effort to develop a market for recovery swaps[14]. When this market reaches larger scale and liquidity, it may provide a benchmark for the recovery rate assumption of individual reference names.

The market norm is to use a 40% recovery assumption

In the credit triangle, default probability interacts with recovery rate assumption and CDS premium in the following ways:

- For a given premium, higher recovery rate assumptions mean higher default probabilities. For example, if one credit has an expected recovery rate of 40% and another credit of 30%, then the loss given default is lower for the former. If both credits trade at the same CDS premium, then the former must have a higher default probability.
- On the other hand, given the same recovery rate assumption, higher premiums obviously mean higher default probabilities.

11 Although they may also reflect factors such as supply and demand, and counterparty risk.

12 The hazard rate in the equation, as explained later in this section, is the default probability per period, conditional on that the reference entity has survived all previous periods. Denoting it as λ and time as t, its relationship with the cumulative default probability is:

$$\text{Cumulative default probability} = 1 - e^{\lambda t}$$

assuming that the premium is continuously compounded.

13 The recovery rates in this report are market prices of debt after default, rather than the ultimate workout value. Defaulted debt prices are more relevant for derivatives pricing.

14 In settlement of a credit event, the buyer in a recovery swap will pay a fixed Recovery Cash Amount; in exchange, the seller will deliver Deliverable Obligations. The buyer will make a profit if the market value of the Deliverable Obligations is higher than the fixed amount that he pays, and vice versa. There is usually no premium payment involved.

The basic equation

In a quantitative CDS model, the probabilities used are based on a term structure of *hazard rates*, which are the probabilities of the reference entity defaulting in each time period, conditional on it having survived all previous periods. These conditional probabilities are then translated into:

- a term structure of *survival probabilities*, which are the probabilities of the reference entity surviving up to the end of each time period[15]; and
- a term structure of non-conditional default probabilities, which is effectively the difference in survival probabilities between two adjacent periods[16].

These probabilities are applied as follows:

- The survival probability for each period is used to weight the contingent premium payment in that period, giving an expected cash flow of the CDS' premium leg.
- The default probability for each period is used to weight the contingent protection payment of 1–Recovery in that period, giving an expected cash flow of the CDS' default leg.

The PV of each of the CDS' two legs is then the discounted present value of each one's expected cash flow. Appendix 1 gives a detailed illustration of these calculations.

For an on-market CDS, the PVs of the two legs are equal. The elements in this equation are:

- the premium;
- the recovery rate assumption;
- the hazard rates (and therefore the survival probabilities and default probabilities);
- and the interest rate curve for discounting.

The hazard rate is the only unknown in the equation, so it can be backed out. To infer a hazard rate curve though, we need an on-market CDS curve, i.e., we need a number of these PV equations at different maturities. Appendix 2 shows the common "bootstrapping" method for deriving a term structure of hazard rates from a quoted CDS curve.

For liquid reference names, there are usually enough market quotes to form such a CDS curve. Where there is only one or two quoted maturities, a flat or interpolated curve is often assumed. Where there are no liquid CDS quotes at all, such as in the case of pricing "points-upfront" CDS for distressed credits, bond prices are often used to infer the hazard rates. The principle for inference is the same as that discussed above, however the implementation needs to take into account the funded nature of bonds, i.e., both the principal and coupon cash flows.

Now that we have obtained a hazard rate curve from the on-market CDS quotes, we can go on to price an off-market CDS, using the same quantitative model and hazard rates, as well as the same recovery assumption and interest rate curve. In the case, CDS pricing is set to re-establish the PV equation of the swap's two legs. Since the pricing is the only unknown element in the equation, it can be calculated (see Appendix 1 for the mathematical formula).

15 For example, the survival probability for period 2
= (1 – the hazard rate for period 1) * (1 – the hazard rate for period 2).
= 1 – the cumulative default probability for period 2

16 For example, this probability for period 2
= the survival probability for period 1 – the survival probability for period 2
= the cumulative default probability for period 2 – the cumulative default probability for period 1

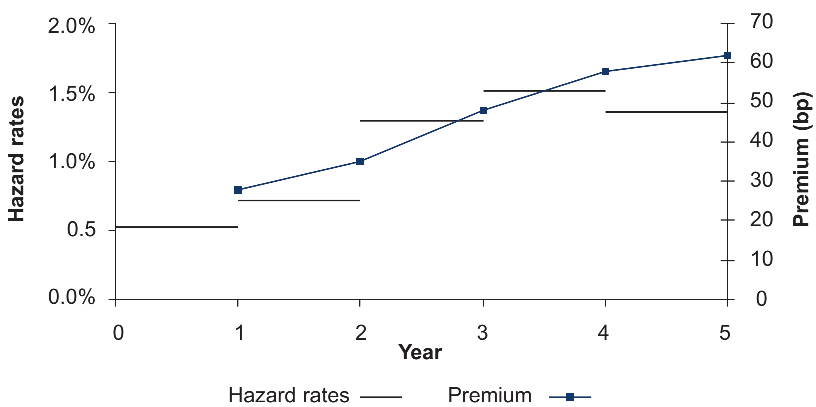
Model assumptions

Certain assumptions are made in the quantitative models. A simple model usually assumes that:

- recovery is a fixed percentage of par, independent of the model and constant over time;
- the interest rate (i.e., the discount rate) and default processes are independent of each other, so that there is no correlation between the level of default and interest rates;
- hazard rates have a defined functional form, usually assumed to be "piecewise constant". This means that hazard rates change from one period to the next, but remain constant within each period (Chart 2).

More sophisticated models may drop these assumptions, and model the joint behaviour of default rates, recovery rates and even interest rates. This will make a difference to the CDS valuation, which is partly why pricing by simple models available publicly can be different from pricing by more sophisticated proprietary models. Nevertheless, in most cases these differences are not very significant. After all, every model is calibrated to the market so that its results fit into the quoted CDS curve.

Chart 2: An example of hazard rates



Source: BNP Paribas

CHAPTER 2: CDS MARK-TO-MARKET VALUATION

1. CALCULATING THE MARK-TO-MARKET VALUE

CDS are spread products, whose mark-to-market value changes as spreads change

As mentioned in the previous chapter, CDS contracts have zero net present value at inception. After inception, market premium levels move for the reference entity, reflecting changes in its credit quality and general market dynamics. These changes in premium cause CDS to have either a positive or negative net present value. For a protection buyer, if the market premium moves wider than the contract premium, he will experience a mark-to-market ("MTM") gain because he bought the protection cheaper than currently available in the market. Vice versa if market premiums tighten. Obviously the protection seller experiences the opposite MTM results.

Assume an offsetting trade, which results in an annuity-like net cash flow stream

Calculating a CDS MTM is essentially the same as calculating the cost of entering into an offsetting transaction. Suppose an investor bought 5-year protection at 100bp per annum, and 1 year later the protection widened to 120bp. The investor would then have a MTM gain.

MTM is the PV of this annuity

To calculate this MTM amount, we can assume a hypothetical offsetting trade where the investor sells protection at 120bp for 4 years, thereby hedging his position. Assume also that the offsetting trade matches the original one perfectly in terms of contract terms, including the same payment and maturity dates, except for the contract premium. This will leave a residual cash flow of 20bp per annum (5bp per quarter) for 4 years in favour of the investor, effectively a 4-year annuity. The present value of this annuity is the MTM amount.

This annuity can terminate early...

However, we need to bear in mind that these cash flows can cease, since the investor will receive the 5bp per quarter only until the earlier of a credit event and contract maturity. Upon the occurrence of a credit event, Credit Event Notices will be served and premium payments will then cease on both transactions, wiping out future annuity payments.

...so the PV is calculated with survival probabilities

Valuing this annuity payment therefore involves more than just discounting it using risk free rates. It also involves weighting the annuity with the probabilities of receiving these quarterly payments, i.e., the probabilities of no credit events occurring before each quarterly payment date. These are the survival probabilities that were introduced in the previous chapter.

So, the MTM of a CDS is the present value of an annuity representing the difference between the contract premium and the current market premium, with the annuity cash flows weighted by the survival probabilities. Mathematically, this can be represented as[17]:

$$MTM=\sum_{i=1}^{n}(S-S')*\Delta_{i-1,i}*(1-q(t_i))*B(0,t_i)$$

n:	the number of (quarterly) annuity payment periods
S:	contract premium (annual)
S':	current market level (annual)
$\Delta_{i-1,i}$:	length of the payment period (one quarter)
$q(t_i)$:	cumulative default probability up to time t_i
$1-q(t_i)$:	survival probability at t_i
$B(0,t_i)$:	discount factor for t_i

17 Strictly speaking, there should also be an accrual factor, representing the expected PV of S-S' from the last premium payment date to the default date (the Event Determination Date). For simplicity, we ignore this factor in the MTM formula.

In the previous example where the CDS premium widens from 100bp to 120bp, the MTM for the protection buyer is 5bp per quarter, weighted by the survival probabilities summarised in Table 1, and discounted to the present. For each EUR 1m notional amount, the resulting MTM gain is EUR 7,322.

Table 1: MTM example

Tenor	Survival probability	Discount factor
0	100.00%	1.0000
3m	99.47%	0.9948
6m	98.97%	0.9890
1yr	97.98%	0.9738
2yr	96.02%	0.9325
3yr	94.11%	0.8855
4yr	92.23%	0.8700

Source: BNP Paribas
Assumptions include:
1. A flat CDS curve of 120bp at the time of MTM.
2. A recovery rate of 40%.

Risky PV01

We can also put the previous equation into the following form[18]:

$$MTM = Annuity * Risky\ PV01$$

with the “risky” PV01 being the sum of the discount factors weighted by their corresponding survival probabilities, i.e., the sum of the risky discount factors. This risky PV01 measures the present value of 1bp risky annuity received or paid until the earlier of a credit event or the maturity of the CDS. The CDS MTM is therefore the annuity multiplied by the PV01.

In the previous example, the PV01 at the “unwinding” spread of 120bp is EUR 366.1. The 20bp widening will result in a MTM change of EUR 366.2*20 = EUR 7,322, which is the same amount as calculated above.

18 For simplicity, we ignore the accrued premium.

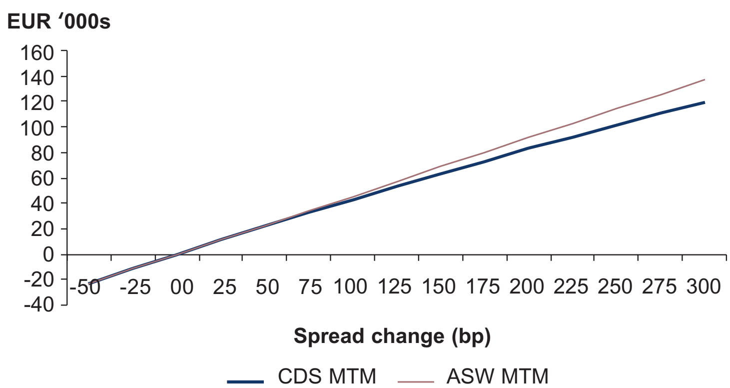
2. CDS MTM SENSITIVITIES

Sensitivity to the premium level

Obviously the bigger the change in premium, the higher the absolute amount of a CDS' MTM. Chart 3 shows the MTM of a 5-year protection position when spreads widen. Although the CDS MTM in this case is an increasing function of spread widening, it is not a straight line and the rate of increase gradually slows down. This is due to the use of survival probabilities in marking-to-market CDS positions. Wider premiums imply higher default probabilities and lower survival probabilities, which reduce the marginal increase in the value of the spread change.

The use of survival probabilities reduces the absolute amount of a CDS' MTM. This effect is more pronounced at higher premium levels

Chart 3: Comparison of CDS and asset swap MTM



Source: BNP Paribas
The MTM amounts are for EUR 1m notional (or principal) of a 5-year long protection (or short credit) position, with an initial premium (or spread) of 100bp and a recovery assumption of 40% for the CDS MTM. The Libor curve is assumed to be unchanged.

Chart 3 also shows that CDS and asset swap MTM amounts differ for the same spread change. This is because the MTM of an asset swap does not involve any use of the bond issuer's survival probabilities. The calculation is therefore more straightforward, and involves simply discounting an annuity stream of the spread change at the Libor curve[19]. The MTM is therefore the spread change times the PV01, which is the sum of the discount factors calculated from the Libor rates. This PV01 is obviously higher than the "risky" PV01 for calculating CDS MTM, which is the sum of the discount factors weighted by the survival probabilities for corresponding periods.

CDS and asset swap MTM amounts differ

The result is that the absolute amount of an asset swap's MTM is always bigger than that of a CDS, given the same spread change. In other words, for a profitable position, CDS will have a smaller gain; for an unprofitable position, CDS will produce a smaller loss. CDS therefore have lower sensitivity to spread movements than comparable asset swaps; they have lower duration.

MTM gains are smaller for a CDS than for an asset swap. And so are MTM losses

19 Assuming AA rated counterparties.

The difference is usually not large when spreads are at relatively low levels, where survival probabilities are high. In the example in Chart 3, even when spreads widen by 100bp, the MTM differential is only EUR 2,560 for a notional of EUR 1m, or less than 0.3% of the notional. However, the difference shows up quite significantly when spreads reach the high yield levels. For a 300bp widening in the example, the MTM differential is EUR 17,400 for every EUR 1m notional, or almost 2% of the notional amount.

For the above comparison, we have assumed that the CDS premium and the asset swap spread widen at the same pace. In reality, this may not be true, especially when bonds start to trade at distressed levels. In these cases, as mentioned in Section 3 of the previous chapter, the low survival probability has an accelerating effect on the premium, making it rise faster than the asset swap spread.

The whole picture, therefore, is: for every 1bp change, the CDS MTM is lower than the asset swap MTM; however, for a deteriorating credit, the CDS premium tends to widen more than the asset swap spread. Whether the combined effects of the two forces produce a higher or lower CDS MTM is case-dependent.

Sensitivity to recovery assumptions

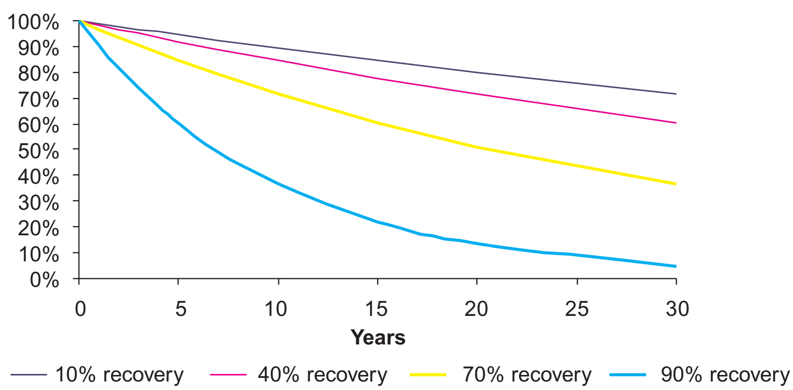
For performing credits, a 40% recovery assumption is usually used for pricing an off-market CDS and calculating its MTM. On the other hand, the recovery assumptions for distressed credits reflect the current market expectation of the recovery value, and therefore can be higher or lower than the 40% mark.

Regardless of whether a “standard” 40% assumption is used, it is important to note that recovery assumptions affect CDS MTM values. This is because, for a given premium, the higher the recovery rate assumption, the higher the default probability and, therefore, the lower the survival probability (see Section 4 of the previous chapter).

Recovery rate assumption drives the shape of the survival probability curve

Chart 4 shows that an assumption of 90% recovery produces substantially lower survival probabilities than does a 10% recovery assumption. This is true for all maturities. In addition, the longer the time horizon, the higher the cumulative default probabilities and the lower the survival probabilities, as evidenced by the downward slopes of all curves in the chart.

Chart 4: Recovery assumptions affect survival probabilities

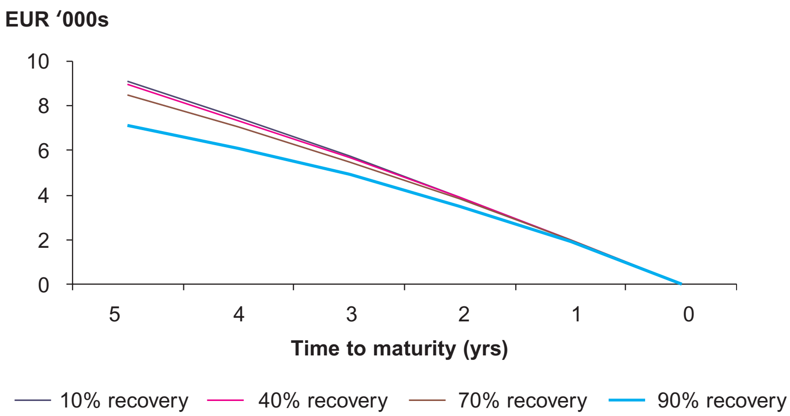


Source: BNP Paribas
A flat premium curve of 100bp is assumed.

Higher recovery assumptions lead to lower MTMs

For CDS MTM, the higher the recovery assumption, the lower the MTM amount given the same change in premium. Chart 5 shows the MTM amounts for a 5-year protection position when the premium widens from 100bp to 120bp, given different recovery rate assumptions. The 90% recovery assumption gives the lowest MTM amounts in the chart, and the 10% recovery the highest MTM. Nonetheless, at relatively low premium levels (120bp in this example), recovery rates in the intermediate range (around 40%) do not cause large differences in the MTM values.

Chart 5: The sensitivity of CDS MTM to recovery assumptions



Source: BNP Paribas
The MTM amounts are for EUR 1m notional of a 5-year long protection position, when premium widens from 100bp to 120bp.

Sensitivity to the remaining time to maturity

Chart 5 also shows that the MTM amounts tend towards zero as the CDS contract draws closer to maturity. Clearly, the shorter the remaining time to maturity, the less the MTM change for a given change of premium.

3. MECHANISMS FOR UNWINDING CDS

MTM changes may prompt a counterparty to unwind a CDS, thereby crystallising a gain or loss. The unwinding mechanism is a key difference between cash bonds and CDS. If a cash investor wants to exit a bond investment, he simply sells the bond. He will then have no residual cash flows from, or contractual obligations to, his trading counterparty. This is not necessarily true for someone unwinding a CDS, as explained below.

There are typically three ways to unwind a CDS:

Three ways to unwind a CDS

Terminating the transaction with the original CDS counterparty

The investor can settle the MTM gain or loss with the original counterparty and terminate the contract. There will be no residual cash flows or exposure to the counterparty's credit risk.

Assigning the contract to a new counterparty

The investor can also assign the contract to a new counterparty, and settle the MTM gain or loss with this new counterparty. This assignment, or "novation" as it is called in the ISDA Definitions, involves the original counterparties (the "Transferor" and the "Remaining Party" in the novation) and the new party (the "Transferee") all agreeing to the transfer of all the Transferor's rights and obligations to the Transferee. Part of this is obviously that the Remaining Party must agree to take on the counterparty credit risk of the Transferee. The Transferor thereby ends his involvement in the transaction.

The 2003 ISDA Credit Derivatives Definitions have a standard Novation Agreement and Novation Confirmation to help counterparties document and obtain the requisite consents to the assignment.

Entering into an offsetting transaction

A CDS counterparty can also "unwind" his existing position by entering into a new offsetting CDS transaction, where he takes the opposite protection position. This offsetting transaction should have virtually the same contract terms as the original CDS regarding the reference entity, the maturity date, etc., except for i) the premium, which is dictated by the prevailing market level; and ii) the tenor, which is the remaining time to maturity of the original CDS.

Strictly speaking, this is hedging rather than an unwind, as it leaves the original position open and creates a new one. It also involves more documentation and additional legal and counterparty credit risk. An investor must ensure that any legal or other basis risk between the two transactions is minimised.

Offsetting also means that the "unwinding" party does not receive or pay the MTM gain or loss upfront. Instead, he will be subject to a prospective net premium cash flow, which is the difference between the premiums of the offsetting transactions. The MTM value of this "unwind" is the expected value of this cash flow, but the realised value can be different, as upon the occurrence of a credit event the cash flow will stop[20].

Because of these constraints, this method is not popular with end-investors. However, offsetting is extensively used by dealers when "unwinding" positions for both clients and themselves, and is part of their dynamic hedging process. Dealers usually hedge their contracts with investors by entering into offsetting transactions in the market. Once an investor unwinds a CDS position, by termination of the contract for example, the dealer also needs to unwind the hedge, often by entering into a new offsetting transaction to the hedge.

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20 An investor can hedge the potential loss of the remaining (unrealised) MTM value. For example, an investor with a MTM gain is long a future contingent annuity stream. He can hedge it by buying a strip of CDS protection on the same reference entity, with the maturity of each contract corresponding to each annuity payment date. The notional amount of each CDS equals the annuity payment / (1 – Recovery), so that the expected payoffs from the protection strip just offset the loss of the annuity payments. An alternative method is to buy protection with one CDS contract to cover all the future annuities, and then dynamically manage the hedge. The initial notional amount of the hedge is set so that the contingent protection payment (1 – Recovery) will offset, or equal, the remaining MTM value. The investor then gradually reduces the notional amount of the hedge as the annuities are realised and the remaining contingent cash flows reduce.

APPENDIX 1: BASIC EQUATIONS IN CDS' QUANTITATIVE PRICING MODELS

The PV of the premium leg is:

- the premium for each payment period times the survival probability for that period;
- this stream of expected premium payments is then discounted at the risk-free rate to give the present value;
- and lastly, the expected PV of the accrued premium from the last payment date to the modelled default date is added. For simplicity, we will ignore this factor.

Mathematically, it can be represented as:

$$PV(premium\ leg) = \sum_{i=1}^{n} S^*\Delta_{i-1,i}^*(1-q(t_i))^*B(0,t_i)$$

S:	premium (annual)
$\Delta_{i-1,i}$:	length of the payment period
$q(t_i)$:	cumulative default probability up to time t_i
$1-q(t_i)$:	survival probability at t_i
$B(0,t_i)$:	discount factor for t_i

The PV of the default leg is:

- the default payment of 1 – Recovery times the probability of default in each period;
- this stream of expected protection payments is then discounted at the risk-free rate to give the present value.

Mathematically, this can be represented as:

$$PV(default\ leg) = \sum_{j=1}^{m} (1-R)^*(q(t_j)-(q(t_{j-1}))^*B(0,t_j)$$

R:	recovery rate
$q(t_j)$-$q(t_j$-1):	default probability in period j

Beginning with the on-market CDS quotes, the PVs of the two legs are equal:

$$\sum_{i=1}^{n} S^*\Delta_{i-1,i}^*(1-q(t_i))^*B(0,t_i) = \sum_{j=1}^{m} (1-R)^*(q(t_j)-q(t_{j-1}))^*B(0,t_j)$$

With a recovery rate assumption and observable values of on-market CDS premiums and the risk-free rates, the only unknowns in the above equation are default probabilities. We can therefore back out a term structure of these probabilities from an on-market CDS curve.

Using the same probabilities, we calculate the off-market CDS premium, S', as the premium to re-establish the PV equation of the swap's two legs:

$$S' = \frac{\sum_{j=1}^{m} (1-R)^*(q(t_j)-q(t_{j-1}))^*B(0,t_j)}{\sum_{i=1}^{n} \Delta_{i-1,i}^*(1-q(t_i))^*B(0,t_i)}$$

APPENDIX 2: BOOTSTRAPPING HAZARD RATES

In Chart 2, there are five on-market CDS quotes for the reference entity, with maturities from 1 to 5 years. With a given risk-free rate term structure and an assumption for the recovery rate, we can bootstrap the hazard rates for year 1 to year 5, using the PV equation of the swap's two legs.

For year 1, this equation is simply:

$$S_1*(1-p_{0,1})*B_{0,1} = (1-R)*p_{0,1}*B_{0,1}$$

S_1:	premium for 1 year
$p_{0,1}$:	hazard rate for year 1
$B_{0,1}$:	discount factor for year 1
R :	recovery rate

Therefore,

$$p_{0,1} = \frac{S_1}{S_1+(1-R)}$$

Now that the hazard rate for year 1 is known, we go on to bootstrap p1,2, the hazard rate for year 2, which is the probability of the reference entity defaulting in year 2, having survived year 1.

Specifically, the probability of receiving the premium in year 2, i.e., the probability of no default in either year (the survival probability), is:

$$(1-p_{0,1})*(1-p_{1,2})$$

And the probability of receiving the default payment in year 2 is:

$$(1-p_{0,1})*p_{1,2}$$

In the following equation, the left hand side sums the PVs of the expected premiums in year 1 and year 2; the right hand side sums the PVs of the expected default payments in year 1 and year 2. Since p1,2 is the only unknown, it can now be "bootstrapped".

$$S_2*(1-p_{0,1})*B_{0,1}+S_2*(1-p_{0,1})*(1-p_{1,2})*B_{0,2}=(1-R)*p_{0,1}*B_{0,1}+ (1-R)*(1-p_{0,1})*p_{1,2}*B_{0,2}$$

S_2:	premium for 2 years
$p_{1,2}$:	hazard rate for year 2
$B_{0,2}$:	discount factor for year 2
R:	recovery rate

With the hazard rates for year 1 and year 2 backed out, we can go on to bootstrap the hazard rate for year 3, and then for year 4 and year 5. We give an example in Table 2.

Table 2: An example of hazard rates and survival probabilities

Year	CDS premium	Hazard rate	Survival probability	Cumulative default probability
1	28bp	0.53%	99.47%	0.53%
2	35bp	0.71%	98.76%	1.24%
3	48bp	1.29%	97.49%	2.51%
4	58bp	1.51%	96.01%	3.99%
5	62bp	1.35%	94.71%	5.29%

Source: BNP Paribas
Recovery rate assumption of 40%

APPENDIX 3: PRICING CDS - A PRACTICAL EXAMPLE

BNP Paribas has developed a number of online tools to assist clients in pricing various credit derivative products. Here we describe the steps used to price a CDS unwind.

Step 1: Log into LiveCredit (www.livecredit.bnpparibas.com). Go to Credit Derivatives > Pricing Tools > Pricer.

Step 2: Select a reference entity in the Trade Details panel. Once a selection is made, its credit curve will be automatically uploaded in the Tenor & Curves panel. Alternatively, fill in a flat curve as agreed with the trade counterparty. This can be easily done by filling in the Flat window at the bottom of the column and check the box next to it.

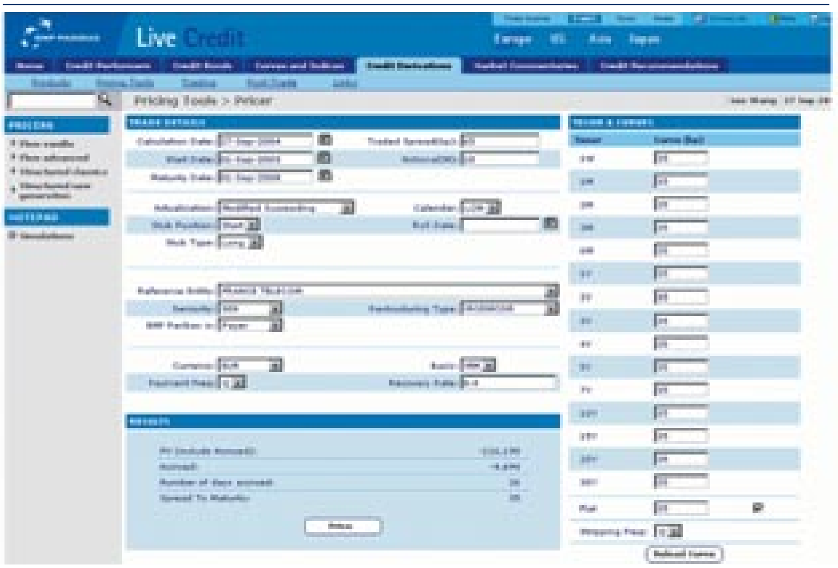
Step 3: Fill in the windows in the Trade Details panel, such as the Start Date, Maturity Date, Traded Spread, Notional, Seniority, Restructuring Type, Currency and Recovery Rate. The market norm is to use a 40% recovery.

Step 4: Press the Price button at the bottom of the page.

Step 5: Check the Results panel. The first line gives the total MTM value, including the accrued premium. This is from the trader's point of view so the investor has the opposite value. The next two lines specify the amount of the accrued premium and the number of days accrued. The last line lists the spread for the remaining maturity, according to the entity's credit curve.

Exhibit 1 shows a hypothetical CDS transaction where an investor sold 5-year protection on France Telecom at 65bp on 1 September, 2003, and is unwinding the trade at 35bp on 27 September, 2004. In this example, the investor's gain is EUR 116,190.

Exhibit 1: An example of pricing a CDS on LiveCredit



Source: BNP Paribas

IMPORTANT DISCLOSURES:

Recommendation System:

Type	Terminology	Horizon
Credit Trend (1)	Positive/ Neutral/ Negative	6 months
Relative Value (2)	Outperform/ Market Perform/ Underperform	1 month
Investment Recommendation (3)	Buy/ Hold/ Reduce/ Sell	Up to 6 months

(1) Credit trend is based on underlying Credit fundamentals, business environment and industry trends;
(2) **Relative Value** is based on expected market performance relative to sector;
(3) **Investment Recommendation** is based on BNPP Credit Trend and Relative Value opinions.
(*) **Buy:** Overweight exposure within industry sector, based on strong financial profile, conservative risk and/or solid relativevalue considerations, outperforming or demonstrating average performance within peer group;
Hold: Exhibits solid to strong credit fundamentals, but average total return characteristics within peer group;
Reduce: Credit exposure should be pared down based on weakening fundamentals and/or below average relative value characteristics within peer group;
Sell: Sell exposure largely based on deteriorating credit fundamentals and/or negative headline or event risk.

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