Stock Return Predictability and Model Uncertainty

By

Doron Avramov*

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*The Robert H. Smith School of Business, University of Maryland, e.mail: davramov@rhsmith.umd.edu, phone: (301) 405-0400, fax: (301) 405-0359. The paper is based on my dissertation at the Wharton School, the University of Pennsylvania. I am grateful to my dissertation committee members, Michael Brandt, Craig MacKinlay, Andrew Metrick, Frank Schorfheide, and especially to my committee chairman Rob Stambaugh for useful comments and discussions. Also acknowledged are the comments of Gurdip Bakshi, Tarun Chordia, Jeremy Gold, Robert Hauswald, Chris Kirby, Anthony Lynch, Lubos Pastor, Nagpurnanand Prabhala, Krishna Ramaswamy, Lucie Tepla, Amir Yaron, and seminar participants at Boston College, Cornell University, Deutsche Bank Asset Management, Goldman Sachs Asset Management, London Business School, University of Maryland, University of Pennsylvania, Vanderbilt University, the 2000 meetings of the Washington Area Finance Association, and the 2000 meetings of the Western Finance Association. All remaining errors are mine.

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Abstract

We use Bayesian model averaging to analyze stock return predictability from a perspective of an investor who faces model uncertainty, or uncertainty about which economic variables should appear in the return forecasting model. Model uncertainty could be more important than the within-model parameter uncertainty, especially when economic variables are at their recently observed levels. The Bayesian approach to model uncertainty is consistent with the existence of out-of-sample predictability in contrast to the classic based analysis, which detects no such predictability. Moreover, the odds in favor of predictability are substantially higher for small-versus-large and high-versus-low book-to-market stocks.

Introduction

Financial economists have identified economic variables that predict aggregate stock returns through time. Such variables include the dividend-price ratio, expected and unexpected inflation, lagged returns, and the differences between yields on long-term and short-term government bonds and between low grade and high grade corporate bonds (e.g., Campbell (2000)). For several reasons, the "correct" specification of the regression of stock returns on predictive variables has remained uncertain. First, asset pricing theories are not explicit about the true predictors, raising doubts about the external validity of the empirical evidence. In particular, several recent studies (e.g., Bossaerts and Hillion (1999)) confirm in-sample predictability but fail to detect out-of-sample predictability. Second, the multiplicity of potential predictors raises difficulties in interpreting the empirical evidence. For example, one may find that an economic variable is significant based on a particular collection of regressors, but becomes insignificant when an alternative specification is examined. Whether such a variable is a robust predictor or not is ambiguous.

The uncertainty about the true set of predictive variables, commonly termed "model uncertainty," is a small sample phenomenon. In sufficiently large samples all potential predictors can be included in an all-inclusive specification. In this regression, irrelevant variables will have slope-coefficient estimates converging to zero, their true value. However, in practice there are many possible predictive variables, but only a limited number of observations. The classical regression paradigm thus offers little help.

The paper undertakes a Bayesian model averaging perspective to analyze the sample evidence about return predictability when the true forecasting model is unknown a priori. The Bayesian procedure computes posterior probabilities for a set of competing return-generating models and uses the probabilities as weights on the individual models to obtain one overall weighted model. The weighted model summarizes the dynamics of future stock returns in the presence of model uncertainty.

Bayesian model averaging contrasts markedly with the traditional approach of model selection, i.e., using a specific criterion to select a single model and then operating as if the model is correct. By implicitly assuming that all models are equally likely a priori, model selection criteria are biased in favor of return predictability. To illustrate, when M economic variables are suspected relevant in predictability there are 2^M competing linear models, all but one retain predictive variables. Therefore, the implied prior-odds ratio against predictability is $\frac{1}{2^M-1}$, approaching zero as M gets large. In our proposed paradigm, the decision-maker has the discretion to elicit prior odds. Moreover, implementing model selection criteria, the econometrician views the selected model as being the 'true' one and discards the other models as worthless, thereby ignoring model uncertainty.[1]

The analysis shows that incorporating model uncertainty into stock return predictability undermines the apparent predictive power of several explanatory variables. Such variables are significant based on the individual forecasting models, but not based on the weighted model, which accounts for model uncertainty. However, the overall in-sample evidence based on posterior-odds ratios and other measures confirms the presence of return predictability. Moreover, in contrast to the evidence based on model selection criteria, which confirms in-sample but not out-of-sample predictability (e.g., Bossaerts and Hillion (1999)), the evidence based on Bayesian model averaging is consistent with out-of-sample predictability.

Our posterior analysis points to several economic variables as useful predictors of monthly and quarterly returns on equity portfolios sorted on size and book-to-market. Such variables

[1]Following this logic, the Bayesian procedure of computing posterior probabilities for all competing models but at the same time conditioning the inference on the single highest-posterior-probability model (e.g., Cremers (2000)) essentially ignores model uncertainty.

include the aggregate measure of earnings yield and the difference between lagged returns on long-term and short-term government bonds. Several other variables possess substantially smaller probabilities of being correlated with future returns. Those variables include the aggregate measures of book-to-market and dividend yield, lagged returns, inflation, and the trend-deviation-in-wealth.

The posterior analysis also detects prominent dispersion in predictability across the size book-to-market portfolios. Holding book-to-market fixed, posterior odds in favor of predictability are substantially higher for small-versus-large capitalization stocks. Controlling for size, the posterior odds are higher for high-versus-low book-to-market stocks. Thus, the most predictable returns are on the smallest size highest book-to-market portfolio. Those results are robust to various prior specifications.

Model uncertainty carries implications for a portfolio-optimizing investor. In particular, the investment environment is represented by a predictive distribution that averages out the uncertainty about the forecasting model and integrates out the uncertainty about the within-model parameters. The predictive analysis shows that the variance of future stock returns attributed to model uncertainty is, on average, more important than its parameter uncertainty counterpart based on monthly observations, but the reverse is true for quarterly observations.

The predictive analysis also shows that the investment opportunity set in the presence of model uncertainty is consistent with the existence of stock return predictability. For example, the asset allocation across a riskfree cash account and six size book-to-market portfolios displays high sensitivity to the values of the predictive variables observed at the time the investment decision is made, even when the investor's prior beliefs are weighted against return predictability.

The remainder of the paper proceeds as follows. Section I derives an analytical result for the posterior probabilities of all the forecasting models. It also derives three measures for investigating the robustness of predictive variables in the presence of model uncertainty. Section II develops an econometric framework to study asset allocation under model uncertainty. Section III describes the sample data, and Section IV contains empirical results. Conclusions and ideas for future research are presented in Section V. All the mathematical derivations are presented in the appendix.

I Predictability in the Presence of Model Uncertainty

When M economic variables belong to the a priori set of stock return predictors there are 2^M competing return-generating specifications. Each of these obeys the form

$$r_t' = x_{j,t-1}' B_j + \epsilon_{j,t}', \tag{1}$$

where r_t is an $N \times 1$ vector of continuously compounded returns on N common stocks in excess of the continuously compounded T-bill rate, j is a model-specific indicator, $x_{j,t-1}' = (1, z_{j,t-1}')$, $z_{j,t-1}$ is a model-unique subset, which contains m variables observed at the end of $t-1$, B_j is an $(m+1) \times N$ matrix of the regression coefficients. The parameter m ranges between zero and M. The former corresponds to the *iid* model, which discards all variables as worthless predictors. The latter corresponds to the all-inclusive specification. We assume that $\epsilon_{j,t}$, the forecast error, is normally distributed with conditional mean zero and variance-covariance matrix Σ_j. The conditional homoskedasticity assumption is for tractability of analysis. This assumption is made in several other studies, including Barberis (2000) and Pastor and Stambaugh (2000).

The study adopts Bayesian model averaging to account for the uncertainty about the true forecasting model. The Bayesian procedure computes posterior probabilities for the collection of all models and uses the probabilities as weights on the individual models to obtain one overall weighted forecasting model, which summarizes the dynamics of future stock returns in the presence of model uncertainty. The posterior probability computation necessitates eliciting prior distributions of all the relevant parameters conditional on each possible model (e.g., Kass and Raftery (1995) and Poirier (1995)).

Our informative prior distribution for each of the model-specific parameters (B_j, Σ_j) is based on an hypothetical prior sample weighted against predictability, as suggested by Kandel and Stambaugh (1996). In that sample, the slope coefficients in the regression of excess stock returns on a set of information variables are equal to zero, and the means and variances of stock returns and predictive variables are equal to the actual sample counterparts, which are given by:

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t, \tag{2}$$

$$\hat{V}_r = \frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})(r_t - \bar{r})', \tag{3}$$

$$\bar{z}_j = \frac{1}{T}\sum_{t=0}^{T-1} z_{j,t}, \tag{4}$$

$$\hat{V}_{j,z} = \frac{1}{T}\sum_{t=0}^{T-1}(z_{j,t} - \bar{z}_j)(z_{j,t} - \bar{z}_j)', \tag{5}$$

where T is the actual sample size.

Using statistics from the actual sample to elicit some of the parameters of the prior distribution is commonly termed "empirical Bayes" (e.g., Maritz and Lwin (1989)). The empirical Bayes procedure is also undertaken by Pastor (2000) and others who implement Bayesian methods to study various applications in financial economics. Based on the hypothetical prior sample, the prior for the regression coefficient B_j conditional on Σ_j is given by the multivariate Normal distribution:

$$\text{vec}(B_j)|\Sigma_j \sim N\left(\text{vec}(B_{j,0}), \frac{1}{T_0}\Sigma_j \otimes \begin{bmatrix} 1+\bar{z}_j'\hat{V}_{j,z}^{-1}\bar{z}_j & -\bar{z}_j'\hat{V}_{j,z}^{-1} \\ -\hat{V}_{j,z}^{-1}\bar{z}_j & \hat{V}_{j,z}^{-1} \end{bmatrix}\right), \tag{6}$$

where $B_{j,0} = [\bar{r}, \mathbf{0}_j]'$, $\mathbf{0}_j$ is an $N \times m$ matrix of zeros reflecting the 'no predictability' prior sample, T_0 is the size of the hypothetical sample, and vec($\bullet$) denotes the vector formed by stacking the successive transformed rows of the matrix. The marginal prior for Σ_j is inverted Wishart (e.g., Zellner (1971))

$$\Sigma_j \sim IW(T_0\hat{V}_r, T_0 - N - 1). \tag{7}$$

Of course, the posterior analysis depends upon T_0, which determines the strength of the informative prior. As an extreme, if T_0 approaches infinity the investor displays dogmatic beliefs about no predictability. Any finite sample size cannot reverse such tight beliefs. Our task is, therfore, to pick a reasonable value for the prior sample size. Kandel and Stambaugh (1996) motivate such a value. Using Monte Carlo simulations, they show that the implied priors of R-squared in the regression of excess stock returns on lagged predictive variables are invariant to the number of predictors if the number of hypothetical data entries per parameter is held fixed (50 observations per parameter) as the number of parameters changes. Our analysis relies primarily on this. Essentially, the hypothetical prior size increases as the model contains more explanatory variables. Therefore, we will denote the prior sample size with the model-specific indicator.

Proposition 1 establishes an analytical result for the marginal likelihood, an input in computing the posterior probability. The marginal likelihood for model j is denoted by $P(D|\mathcal{M}_j)$, where D stands for the sample data, described in Section III. For the marginal likelihood computation, D is restricted to include only stock returns, but not predictive variables. This assumption, which will be relaxed as the work proceeds, is consistent with other studies computing marginal likelihood (e.g., Kass and Raftery (1995)) and the traditional model selection criteria (e.g., Bossaerts and Hillion (1999)).

Proposition 1 *The log marginal likelihood of any entertained model, excluding the iid model, is given by:*

$$\begin{aligned}\ln\left[P(D|\mathcal{M}_j)\right] &= -\frac{TN}{2}\ln(\pi)+\frac{T_{j,0}-N-1}{2}\ln|T_{j,0}\hat{V}_r|-\frac{T_j^*-N-1}{2}\ln|\tilde{S}_j| \\ &- \sum_{i=1}^{N}\ln\left\{\Gamma\left(\frac{T_{j,0}-N-i}{2}\right)\right\}+\sum_{i=1}^{N}\ln\left\{\Gamma\left(\frac{T_j^*-N-i}{2}\right)\right\},\end{aligned}$$

where

$$\begin{aligned}\tilde{S}_j &= T_j^*\left(\hat{V}_r+\bar{r}\bar{r}'\right)-\frac{T}{T_j^*}\left(T_{j,0}[\bar{r},\bar{r}\bar{z}_j']+R'X_j\right)\left(X_j'X_j\right)^{-1}\left(T_{j,0}[\bar{r},\bar{r}\bar{z}_j']'+X_j'R\right), \\ X_j &= [x_{j,0},x_{j,1},\dots,x_{j,T-1}]', \\ R &= [r_1,r_2,\dots,r_T]',\end{aligned}$$

$T_j^*=T+T_{j,0}$, *$\Gamma(y)$ stands for the Gamma function evaluated at y, and $|x|$ is the determinant of x. For the iid model $\tilde{S}_{iid}=T_{iid}^*\hat{V}_r$.*

Multiplying the marginal likelihood by the prior probability $P(\mathcal{M}_j)$, which is at the discretion of the decision-maker, and normalizing the resulting quantity produce the posterior probability in favor of the model

$$P(\mathcal{M}_j|D)=\frac{P(D|\mathcal{M}_j)\,P(\mathcal{M}_j)}{\sum_{i=1}^{2^M}P(D|\mathcal{M}_i)\,P(\mathcal{M}_i)}. \quad (8)$$

Having posterior probabilities at hands, the study examines three measures to investigate the statistical robustness of explanatory variables in predictive regressions.

The first is cumulative posterior probabilities of the predictive variables. It is computed as $\mathcal{A}'\mathcal{P}$, where $\mathcal{A}$ is a $2^M\times M$ matrix representing all forecasting models by zeros and ones, designating exclusions and inclusions of predictors, respectively, and $\mathcal{P}$ is a $2^M\times 1$ vector containing model posterior probabilities. The resulting quantity indicates the probabilities that each of the predictive variables appears in the weighted forecasting model. To illustrate, in one polar scenario in which the iid model receives a posterior probability equal to unity the cumulative posterior probabilities are represented by an $M \times 1$ vector of zeros. In the opposite extreme in which the all-inclusive model receives the entire posterior mass the posterior probabilities are represented by an $M \times 1$ vector of ones.

The second measure is a posterior t ratio obtained by dividing the posterior mean of each of the slope coefficients in the weighted model by its corresponding posterior standard error. Focusing on a multiple regression run separately for any risky asset, the posterior mean and variance are given by:[2]

$$\mathsf{E}(B|D) = \sum_{j=1}^{2^M} P(\mathcal{M}_j|D)\,\tilde{B}_j, \tag{9}$$

$$\mathrm{Var}(B|D) = \sum_{j=1}^{2^M} P(\mathcal{M}_j|D)\left\{\frac{T\tilde{S}_j(X_j'X_j)^{-1}}{T_j^*(T_j^*-4)} + \left[\tilde{B}_j - \mathsf{E}(B|D)\right]\left[\tilde{B}_j - \mathsf{E}(B|D)\right]'\right\} \tag{10}$$

where

$$\tilde{B}_j = \frac{T}{T_j^*}(X_j'X_j)^{-1}(T_{j,0}[\bar{r}, \bar{r}\bar{z}_j']' + X_j'R), \text{ for } j = 1 \ldots 2^M \text{ and } j \neq \text{iid},$$

$$\tilde{B}_{iid} = \bar{r}.$$

The mean (9) follows by iterated expectations, conditioning first on the model space. The variance (10) follows by using properties of the inverted Wishart distribution and variance decomposition. The posterior mean is merely a weighted average of slope estimates. The posterior variance incorporates both the estimated variances in every entertained model and the model-uncertainty component attributed to the dispersion in the posterior mean of

[2] It should be noted that $\tilde{B}_j$ and $\tilde{S}$ are of equal dimension for any entertained model since slope coefficients of excluded variables and their variances and covariances with other slope coefficients are zero. To illustrate, we rewrite $\tilde{B}_{iid}$ as $[\bar{r}', 0]'$, where 0 is a $1 \times M$ vector of zeros.

the slope coefficients across the models. Of course, the larger the dispersion, or the greater the ex post uncertainty about the true predictors, the smaller the posterior t-ratio.

It should be noted that in the traditional regression paradigm, one may find that an economic variable is significant based on a particular collection of regressors, but becomes insignificant when an alternative specification is examined. Whether such a variable is a robust predictor or not is ambiguous. The Bayesian approach to model uncertainty implies that such a dispersion in the slope coefficients is reflected through a higher standard error of the coefficient in the weighted model. The robustness of the predictor can be examined based on both the posterior t-ratio and cumulative posterior probability.

The third measure is a posterior-odds ratio obtained by dividing the sum of posterior probabilities assigned to $2^M - 1$ models that retain at least one predictor by the posterior probability of the iid model. Computing posterior odds in financial economics goes back to Shanken (1987) who implements a Bayesian approach to testing portfolio efficiency. Shanken (1987) shows that using posterior odds leads to a particular inference about mean variance efficiency that could differ from the one obtained by the classical p value.

II Model Uncertainty and the Investment Environment

Kandel and Stambaugh (1996), Stambaugh (1999), and Barberis (2000) have shown that predictive regressions are useful in making asset allocation decisions when investment opportunities are time varying. Those studies incorporate estimation risk, but not model risk. This section develops a framework for analyzing investment decisions under model uncertainty. Asset allocations are derived to deliver an economic based metric for gauging the evidence on stock return predictability under model uncertainty. The perceived investment opportunities based on the weighted forecasting model are reflected through the Bayesian weighted predictive distribution.

A The Bayesian Weighted Predictive Distribution

Let $y'_{j,t} = (r'_t, z'_{j,t})$ be the data-generating process corresponding to model j. We assume that the evolution of $y_{j,t}$ is governed by the stochastic process

$$y'_{j,t} = x'_{j,t-1}\Phi_j + u'_{j,t}, \tag{11}$$

where Φ_j is an $(m+1)\times(N+m)$ matrix of regression coefficients and $u_{j,t}$ is an $(N+m)\times 1$ vector of forecast errors.[3] We assume that $u_{j,t} \sim iid \quad N(0,\Psi_j)$. Implied in the data-generating process (11) is a first order VAR for the dynamics of the predictive variables

$$z'_{j,t} = a'_j + z'_{j,t-1}A_j + \eta'_{j,t}. \tag{12}$$

The matrix A'_j is known as the companion matrix of the VAR. The assumption that the VAR is first order is not restrictive since higher-order VAR can always be rewritten in first order form, as discussed by Campbell and Shiller (1988a).

The Bayesian weighted predictive distribution of cumulative excess continously compounded returns averages over the model space and integrates over the posterior distribution that summarizes the within-model uncertainty about Φ_j and Ψ_j. It is given by

$$P(R_{T+K}|D) = \sum_{j=1}^{2^M} P(\mathcal{M}_j|D) \int_{\Psi_j}\int_{\Phi_j} P(\Phi_j,\Psi_j|\mathcal{M}_j,D)\, P(R_{T+K}|\mathcal{M}_j,\Phi_j,\Psi_j,D)\, d\Phi_j d\Psi_j, \tag{13}$$

[3]Equation (11) provides some reasoning for why the marginal likelihood is computed assuming that the data include stock returns only. The marginal likelihood $P(D|\mathcal{M}_j)$ indicates the probability that the data, D, are generated by model j. Obviously, the left-hand-side data in (11) differ across models. Therefore, to compute the marginal likelihood, or any model selection criteria, one would rely on the data generating process in (1) in which the left-hand-side is not model specific.

where K is the investment horizon and $R_{T+K} = \sum_{k=1}^{K} r_{T+k}$. To the best of our knowledge, an analytical solution for the integral in (13) when $K > 1$ is not feasible. Instead, Monte Carlo integration is used. Specifically, sampling from the Bayesian weighted predictive distribution is obtained by first drawing from the distribution of models. Then, the model-specific parameters Φ_j and Ψ_j are drawn from their joint posterior distribution, solved in the appendix. Last, given Φ_j and Ψ_j, an $N \times 1$ random vector of cumulative excess continously compounded returns is drawn from the conditional density of future stock returns described in Proposition 2.

Proposition 2 *The distribution of future stock returns conditioned upon the model, its specific parameters Φ_j and Ψ_j, and the sample data is given by:*

$$R_{T+K}|\mathcal{M}_j, \Phi_j, \Psi_j, D \sim N\left(\lambda_j, \Upsilon_j\right),$$

where

$$\begin{aligned}
\lambda_j &= Kb_j + C_j\left[((A_j')^K - I_m)(A_j' - I_m)^{-1}\right] z_{j,T}, \\
&+ C_j\left[A_j'\left((A_j')^{K-1} - I_m\right)(A_j' - I_m)^{-1} - (K-1)I_m\right](A_j' - I_m)^{-1} a_j, \\
\Upsilon_j &= K\Sigma_j + \sum_{k=1}^{K} \delta_j(k)\Theta_j\delta_j(k)' + \sum_{k=1}^{K} \Lambda_j\delta_j(k)' + \sum_{k=1}^{K} \delta_j(k)\Lambda_j', \\
\delta_j(k) &= b_j'\left[\left((A_j')^{k-1} - I_m\right)(A_j' - I_m)^{-1}\right],
\end{aligned}$$

b_j and C_j are partitions of B_j corresponding to the intercept and slope coefficients in the regression of excess returns on lagged predictive variables, $B_j = [b_j, C_j]'$, and Λ_j and Θ_j are partitions of the variance-covariance matrix Ψ_j:

$$\Psi_j = \begin{bmatrix} \Sigma_j & \Lambda_j \\ \Lambda_j' & \Theta_j \end{bmatrix}.$$

Note that when investors do know the model and its specific parameters the only information from the sample relevant to drawing from the distribution of future stock returns would be the most recent observation of the predictive variables. Also note that no predictability corresponds to $C_{iid} = 0$, which yields $\lambda_{iid} = Kb_{iid}$ and $\Upsilon_{iid} = K\Sigma_{iid}$. Obviously, without accounting for estimation risk, the conditional mean and variance in an iid world increase linearly with the investment horizon. The classical approach employs the conditional distribution of returns to derive asset allocations, thereby assuming normally distributed future stock returns. Accounting for both estimation and model risks, the perceived distribution of future returns departs from normality and may have higher moment features, such as skewness and fat tails.

Of course, in the presence of model uncertainty investment in stocks appears riskier. In particular, based on the weighted predictive distribution the variance of future returns over the investment horizon can be decomposed with respect to both the model space and parameter space. Conducting such a variance decomposition, we show that the variance is attributed to three sources: i) model uncertainty; ii) a mixture of the within-model parameter uncertainty; and iii) a mixture of the within-model forecast error:

$$\mathrm{var}\{R_{T+K}|D\} = \sum_{j=1}^{2^M} P\left(\mathcal{M}_j|D\right)\left[\mathsf{E}\{\Upsilon_j\} + \mathrm{var}\{\lambda_j\} + \left(\tilde{\lambda} - \mathsf{E}\{\lambda_j\}\right)\left(\tilde{\lambda} - \mathsf{E}\{\lambda_j\}\right)'\right], \quad (14)$$

where $\sum_{j=1}^{2^M} P\left(\mathcal{M}_j|D\right)\left(\tilde{\lambda} - \mathsf{E}\{\lambda_j\}\right)\left(\tilde{\lambda} - \mathsf{E}\{\lambda_j\}\right)'$ is the model uncertainty component, $\mathrm{var}\{\lambda_j\}$ is the parameter uncertainty corresponding to model j, and $\tilde{\lambda}$ is the predicted mean of cumulative stock returns that takes account of model uncertainty. The predicted mean is given by

$$\tilde{\lambda} = \sum_{j=1}^{2^M} P\left(\mathcal{M}_j|D\right)\mathsf{E}\{\lambda_j\}. \quad (15)$$

B Portfolio Choice in the Presence of Model Uncertainty

What are the implications of model uncertainty for investment decisions? The optimization problem of a buy-and-hold investor with iso-elastic preferences who allocates funds across N risky assets and the risk-free Treasury bill and who does not know the a priori true set of predictors is given by:

$$\omega^* = \arg\max_{\omega} \int_{R_{T+K}} \frac{\left[(1-\omega'\iota_N)\exp(r_f K) + \omega' \exp(r_f K \iota_N + R_{T+K})\right]^{1-\gamma}}{1-\gamma} P\left(R_{T+K}|D\right) dR_{T+K}, \quad (16)$$

where the integral is taken over the Bayesian weighted predictive distribution, γ is the relative risk aversion parameter, ω is an $N \times 1$ vector denoting portfolio weights chosen for N risky assets at time T, ι_N is an $N \times 1$ vector of ones, and r_f is the continuously compounded risk-free rate of return, assumed constant over the investment horizon. Portfolio weights are restricted to the unit interval, meaning that short selling and buying on margin are precluded; otherwise, the expected utility would be equal to $-\infty$, as explained by Barberis (2000), among others.

The expected utility maximization displayed in (16) is a version of the general Bayesian control problem developed by Zellner and Chetty (1965). Bawa, Brown, and Klein (1979), Jobson and Korkie (1980), Frost and Savarino (1986), Pastor (2000), and Pastor and Stambaugh (2000) compute optimal portfolios in a one-period framework in which returns are assumed iid. Kandel and Stambaugh (1996), Barberis (2000), and Tamayo (2000) analyze a portfolio decision when the investor instead uses a model in which returns can possess predictability. In these studies the conditional distribution of stock returns is integrated over the parameter space to account for estimation risk. Integrating over both the model space and the within-model parameter space is novel in the context of asset allocation.

The integral in equation (16) is approximated by generating independent draws for

$\left\{R_{T+K}^{(g)}\right\}_{g=1}^{G}$ from the weighted predictive distribution using the algorithm described above. A constrained optimization code is then used to maximize the quantity

$$\mathsf{E}\left[U(W_{T+K}(\omega))\right] = \frac{1}{G}\sum_{g=1}^{G} \frac{\left\{(1-\omega'\iota_N)\exp(r_f K) + \omega' \exp(r_f K \iota_N + R_{T+K}^{(g)})\right\}^{1-\gamma}}{1-\gamma} \tag{17}$$

subject to ω being non negative, where G denotes the number of draws.

III Data

The empirical examination uses monthly observations on stock returns and information variables spanning 549 months from April 1953 to December 1998. Also examined is a quarterly sample spanning the same time period. The investment universe consists of the six portfolios formed originally by Fama and French (1993) as the intersections of two size (S,B) and three book-to-market (L,M,H) groups.

We consider the following $M = 14$ information variables: dividend yield on the value weighted NYSE index (Div); book-to-market (BM) on the Standard & Poor's Industrials; earnings yield on the Standard & Poor's Composite index (EY); the winners-minus-losers (WML) one-year momentum in stock returns; default risk spread, formed as the difference in annualized yields of Moody's Baa and Aaa rated bonds (Def); the monthly rate of a three-month Treasury bill (Tbill); excess return on the CRSP value weighted index with dividends (Ret); default risk premium, formed as the difference between the return on long-term corporate bonds and the return on long-term government bond (DEF); term structure premium, formed as the difference between the monthly return on long-term government bond and the one month Treasury bill rate (TERM); January Dummy (Jan); inflation rate (Inf); size premium (SMB); value premium (HML); and term structure slope, formed as the difference in annualized yield of ten-year and one-year Treasury bills (Term). None of the predictors listed above uses information that would not have been available at the time future excess stock returns were predicted.

Data used to compute Div, Tbill, and Ret are from the Center for Research in Security Prices (CRSP) at the University of Chicago. Inputs for calculating book-to-market are obtained from the Standard & Poor's publication: "Security Price Index Record - Statistical Service." Inputs for computing Def are obtained form Citibase. Data on TERM and PREM are from Ibbotson and associates.[4]

In deciding which predictors to include, attention was given to those variables found important in previous studies as well as those popular business cycle variables for which there exist some theoretical "stories." Studies using subsets of the above-listed predictors include Bossaerts and Green (1989), Brandt (1999), Brandt and Ait-Sahalia (2000), Campbell (1987), Campbell and Shiller (1988a, 1988b), Carhart (1997), Chen, Roll, and Ross (1986), Fama and French (1988, 1989, 1993), Fama and Schwert (1977), Ferson and Harvey (1991, 1999), French, Schwert, and Stambaugh (1987), Keim and Stambaugh (1986), Kirby (1997, 1998), Kothari and Shanken (1997), Hodrick (1992), Lo and MacKinlay (1997), Pesaran and Timmermann (1995), Pontiff and Schall (1998), Schwert (1990), and Shanken (1990).

The reasoning for including the variables PREM, TERM, HML, and SMB, mostly notable as economy-wide factors in asset pricing models, follows from Merton (1973) whose

[4] I am grateful to Kenneth French for generously providing returns on size book-to-market portfolios, size premium, and value premium. The winners minus losers portfolio is courtesy of Mark Carhart. Earnings and inflation data were downloaded from Robert Shiller's web-site (http://www.econ.yale.edu/ shiller/data.htm). Earnings yield is formed by dividing the most recent twelve-month earnings by the contemporaneous value of the S&P 500 index. Treasury-bill yields for various maturities are available at the Federal Reserve Board's web-site ((http://www.federalreserve.gov/releases/H15/data.htm).

intertemporal CAPM does not distinguish between variables that predict the market returns and variables that explain the cross-section variation in expected return. Moreover, Liew and Vassalou (2000) show that SMB and HML are useful in predicting economic growth even in the presence of the traditional business cycle variables, making the inclusion of these variables of interest while examining predictability in stock returns.

Table 1 presents summary statistics for the predictive variables (excluding January Dummy) and monthly returns on the six size book-to-market portfolios. We show that dividend yield, book-to-market, earnings yield, default spread, Treasury-bill rate, and term-structure slope display persistence, whereas WML, excess return, default risk premium, term-structure premium, inflation, size premium, and value premium possess lower or no autocorrelation. Also reported (Table 2) are slope coefficients and their corresponding t-ratios obtained by regressing excess returns on each of the size book-to-market portfolios on an intercept and lagged predictive variables described above. A closer look at Table 2 reveals ample evidence for in-sample predictability, as many of the t-ratios exceed two.

IV Results

A The Robustness of Predictive Variables in the Weighted Model

1 The Case of Monthly Observations

Consideration of all linear data-generating processes in the presence of fourteen predictive variables necessitates the comparison of $2^{14} = 16,384$ models. Proposition 1 computes the marginal likelihood for every model, and equation (8) weights the marginal likelihood by the model prior probability and normalizes the result to obtain the model posterior probability. It is assumed throughout that the prior odds of predictability versus no predictability is unity. It is further assumed that the prior probabilities of all the models that include predictors are equal, i.e., such a prior probability is equal to $\frac{0.5}{2^{14}-1}$.

Table 3 reports results. The top figures denote the highest-posterior-probability compositions represented by combinations of zeros and ones designating exclusions and inclusions of predictive variables, respectively. The bottom figures display cumulative posterior probabilities $\mathcal{A}'\mathcal{P}$ for the fourteen predictors, as noted earlier. Several features of the results merit closer attention. The highest-cumulative-probability predictors are the term-structure premium, January Dummy, Treasury bill rate, earnings yield, and inflation. Interestingly, January Dummy appears in all highest-posterior-probability models corresponding to small stocks. This is consistent with Blume and Stambaugh (1983) and Keim (1983), who trace much of the evidence on the size effect to the month of January. Among the traditional market multipliers, i.e., dividend yield, book-to-market, and earnings yield, the latter appears in all the highest-posterior-probability compositions and receives the highest cumulative probabilities. Interestingly, although SMB and HML are reported robust in predicting contemporaneous stock returns (Fama and French (1993)) and future economic growth (Liew and Vassalou (2000)), both are correlated only marginally with future monthly stock returns.

Table 4 exhibits the posterior means of slope coefficients in the weighted model (top figures), as computed in (9), and two t-ratios. The first (middle figures) is obtained by dividing the posterior mean by the posterior standard error corresponding to the first component in (10), thereby ignoring model uncertainty. The second, the posterior t-ratio, (bottom figures) divides the posterior mean by the two sources of uncertainty, including model uncertainty that summarizes the dispersion in the posterior means of slope coefficients across the models.

The extra variance of the slope coefficients in predictive regressions attributed to model uncertainty calls into question the apparent predictive power of several economic variables. Focusing on t-ratios greater in absolute value than two, it appears that the predictive variables Treasury bill rate and term-structure premium are significant based on t-ratios that ignore model uncertainty, but not based upon the posterior t-ratio. In contrast, January Dummy remains significant under both specifications.

Intuitively, the cumulative posterior probabilities should be related somewhat to the posterior t-ratios, and they are. For example, high cumulative posterior probabilities for Treasury bill, earnings yield, and term-structure premium (Table 3) are followed by higher values of posterior t-ratios (Table 4). However, in some cases the absolute values of these measures seem incongruous. As an extreme example, the t-statistic of the dividend yield for the SL portfolio is 0.12, meaning that based on a traditional hypothesis testing dividend yield does not predict (statistically) future returns at any reasonable significance level. However, the cumulative posterior probability of dividend yield is 45%, suggesting some predictive power. Such an apparent contradiction is also documented by Shanken (1987). He shows that based on p-values one fails to reject portfolio efficiency, whereas the odds analysis provides evidence to the contrary.

The third measure undertaken to assess the sample evidence on predictability is the posterior-odds ratio. Based on a prior sample equivalent to 50 observations (or 4 years and two months of hypothetical data) per parameter, we obtain extremely large posterior odds in favor of predictability for every equity portfolio. To examine how strong the prior beliefs against predictability should be to offset the actual sample evidence, we compute odds under various prior specifications. In particular, the prior sample size ranges between 20 and 1,280 years of hypothetical observations per parameter.

Table 5 exhibits results. The analysis shows that also a prior sample size equivalent to 1,280 years of hypothetical observations per parameter is not sufficient to reverse the evidence in favor of predictability. That is, investors *must* form prior beliefs based on a particularly large hypothetical sample weighted against predictability to offset the evidence in favor of predictability, as appears in the actual sample.

Cross-sectional dispersion in predictability is apparent for the various prior specifications. In particular, holding book-to-market fixed, posterior odds in favor of predictability are substantially higher for small-versus-large capitalization stocks. Similarly, controlling for size the posterior odds are higher for high-versus-low book-to-market stocks. In turn, the evidence in favor of predictability is the strongest for the smallest size, highest book-to-market portfolio (SH).

2 The Case of Quarterly Observations

In a recent study, Lettau and Ludvigson (2000) (henceforth LL) introduce the trend-deviation-in-wealth (henceforth TDW) as a powerful predictor of equity markets at short and intermediate horizons. Drawing on the forward-looking model of Campbell and Shiller (1988a), LL argue that TDW summarizes expectations about future stock returns. Trend-deviation-in-wealth is computed as $c_t - wa_t - (1-w)y_t$, where c_t, a_t, and y_t denote log consumption, non-human wealth, and labor income, respectively and w equals the average share of non-human wealth in total wealth. Consumption, wealth, and income data are released by the Federal Reserve Board within two months of the end of a quarter, suggesting that the TDW realization at quarter t is made known to capital market participants at the subsequent quarter and hence must be used to predict returns realized at or after quarter $t+2$.

To examine the predictive power of TDW and the overall evidence about predictability using quarterly observations, an additional set of information variables is constructed with TDW replacing January Dummy.[5] Using quarterly observations and at the same time leaving the prior sample size, T_0, unchanged amount to weighting the prior sample against predictability to a stronger degree as the ratio $\frac{T_0}{T}$ increases three times. To maintain the ratio $\frac{T_0}{T}$ fixed across the monthly and quarterly experiments, posterior probabilities for the new model space are computed with T_0 taking values equivalent to 17 prior observations per parameter.

Table 6 exhibits cumulative posterior probabilities for the new set of predictors. We show that TDW indeed dominates dividend yield, lagged excess return, default-risk spread, and term-structure spread, predictive variables used by LL. (Two predictors examined by LL, dividend-payout ratio and relative bill rate, are not presented in Table 6, but were examined along with the other variables and found not particularly robust in a posterior probability analysis.) TDW outperforms book-to-market, WML, HML, and inflation as well. However, several other variables not accounted for by LL, including term-structure premium, default-risk premium, the three-month Treasury-bill rate, size premium, and earnings yield, possess stronger power in forecasting quarterly returns on all equity portfolios for every entertained prior sample size. Interestingly, SMB appears robust in forecasting quarterly returns on large capitalization stocks, whereas HML is not identified with the highest-posterior-probability models for quarterly observations as well.

Table 7 exhibits t-ratios unadjusted (top figures) and adjusted (bottom figures) to ac-

[5] I thank Martin Lettau for providing data on TDW. It should be emphasized that w, the share of non-human wealth in total wealth, is computed based on all the sample containing data realized after the time future returns were predicted. LL recompute TDW using out-of-sample estimation, which, in turn, produces a fairly small sample, not sufficient to be included here.

count for model uncertainty. Model uncertainty questions the relevance of several explanatory variables. The variables Treasury bill rate and SMB are, in some cases, significant in forecasting quarterly returns based on t-ratios that ignore model uncertainty, but not when such uncertainty is accounted for. In most cases, the t-ratio corresponding to the TDW is smaller (in absolute value) than those corresponding to the variables earnings yield, term-structure premium, Treasury bill rate, and SMB.

3 Bayesian Model Averaging: External Validation

The analysis provides strong evidence that monthly and quarterly returns on portfolios sorted on size and book-to-market are predictable, even when prior beliefs are weighted against predictability. In a related study, Bossaerts and Hillion (1999) confirm the presence of predictability using several model selection criteria. However, they discover that those criteria perform poorly out of sample. This section compares the out-of-sample performance of the weighted forecasting model with that of six other models. The first is an all-inclusive model. The second is the iid model, which rules out return predictability. The other four models are selected using the criteria AIC, SIC, FIC, and PIC, all of which are discussed by Bossaerts and Hillion (1999). Due to the high dimensionality of the model space, the out-of-sample examination focuses on a single risky asset, the value weighted CRSP index encompassing securities traded in NYSE, AMEX, and NASDAQ.

Out-of-sample forecast errors are computed using the following algorithm. Based upon the initial $t = \frac{T}{3}$ sample observations, we compute posterior probabilities for all 2^M compositions and select four models based on the aforementioned criteria. Next, we project the time $t+1$ excess return for each of the seven specifications and retain the corresponding forecast errors. The excess return and predictive variables realized at time $t+1$ are then added to the data set to revise both the model selection and posterior probability computation and to project the time $t+2$ excess return. Following these steps, we obtain $2 \times \frac{T}{3}$ out-of-sample forecast errors for each specification. Table 8 reports the sum of squared forecast errors (SSE), the sum of forecast errors (SFE), and the standard deviation of forecast errors (SDE).

Focusing on the out-of-sample performance of the iid model and the four models selected by AIC, SIC, FIC, and PIC, the evidence shows no out-of-sample predictability. In particular, the SSE's for the iid model are 0.7886 and 0.9492 based on the monthly and quarterly samples, respectively. A similar quantity for the optimally selected models ranges between 0.8010 and 0.8270 based on the monthly sample, and between 0.9966 and 1.1073 based on the quarterly counterpart. Moreover, the absolute value of the sum of forecast errors corresponding to the iid model is 0.3488, whereas the counterpart quantity for the optimally selected models is bounded below by 0.4572. The poor out-of-sample performance of model selection criteria is consistent with Bossaerts and Hillion (1999).

Focusing on all seven specifications, we find that Bayesian model averaging has a superior out-of-sample performance. For example, its SSE and SFE are 0.7793 and -0.0224, respectively, based on the monthly sample. The corresponding quantities based on the quarterly counterpart are 0.9314 and 0.1760. Moreover, the weighted model possesses the smallest dispersion in forecast errors. The overall evidence is thus consistent with out-of-sample predictability.

In a related study, Cremers (2000) conditions his analysis on the highest-posterior-probability model and reports no in-sample and out-of-sample predictability. Our analysis is not conditioned on a single selected model, but rather on the weighted model, which averages over all models under consideration. Conditioning results on a single selected model amounts to ignoring model uncertainty. In particular, when the model space contains as many as 16,384 compositions of predictive variables, the highest posterior probability composition accounts, at least in our analysis, for less than a single percent from the total posterior probability. Focusing on that particular model, one ignores the other 16, 383 competing models that account altogether for around 99% of the posterior mass.

B Model Uncertainty: Implications for the Investment Opportunity Set

We first perform the variance decomposition of future stock returns into the three components, i.e., model risk, estimation risk, and uncertainty attributed to forecast errors. The decomposition is based on the actual end-of-sample realizations. We find that for a single-period investor, the average (across portfolios) contributions of the three components to the overall uncertainty about predicted stock returns are 93%, 3%, and 4%, respectively, based on monthly observations. However, such contributions based on quarterly observations are 79%, 17%, and 4%, respectively. That is, model uncertainty dominates parameter uncertainty based on monthly but not quarterly observations.

Interestingly, focusing on monthly observations Pastor and Stambaugh (1999) show that uncertainty about which pricing model to use is less important, on average, than within-model parameter uncertainty. One of the major differences between the studies is that our setting accommodates information variables. In particular, it is apparent from equation (15) that model uncertainty becomes more prominent with a greater dispersion of the forecasted conditional expected returns across the models. Such a dispersion positively depends upon the deviation of the most recent values of the predictive variables from their historical means. As an extreme example, if such recent values are equal to their historical means, the conditional expected returns are identical across models.

At the end-of-sample period the current values of variables that are perceived to have been indicators of fundamental values, such as book-to-market, dividend yield, and earnings yield, deviate substantially from their sample means, giving rise to the greater impact of model uncertainty. Some figures are presented below:

Predictive Variable	Level as of December 31, 1998	Sample Moments	
		Mean	StDev
BM	0.1178	0.5078	0.1790
Div	0.0155	0.0363	0.0094
EY	0.3907	0.8531	0.2936

What are the implications of the sample size and investment horizon for model-versus-parameter risks?

Higher frequency data provides substantially more information about the variance, but only little additional information about expected returns. Therefore, parameter uncertainty is more prominent based on the quarterly sample. In contrast, with a smaller sample size, model uncertainty, which is merely the dispersion in expected returns across all forecasting models, is affected only marginally.

Parameter uncertainty increases with the investment horizon, as shown by Barberis (2000). However, in longer horizons, predictive variables revert to their long-run means (see autocorrelation coefficients for various lags in Table 1), making conditional expected stock returns look similar across the various forecasting models. We, therefore, expect that the total predictive variance attributed to model uncertainty will converge to a fixed quantity and, consequently, the annualized predictive variance, obtained by dividing the fixed quantity by the horizon length, will diminish with an increasing horizon. The one-period investment horizon thus gives a lower bound on the ratio obtained by dividing parameter uncertainty by model uncertainty.

We next turn to a portfolio selection analysis. Table 9 exhibits asset allocation under model uncertainty across the six size book-to-market portfolios for both monthly (Panel A) and quarterly (Panel B) observations. Asset allocations are derived based on two scenarios, in which the recent values of predictive variables are equal to the actual end-of-sample realizations and to the sample means. The investment horizon ranges between one and ten years. The relative risk-aversion coefficient takes the values three, six, and nine. Also reported are total allocations to equities and a certainty equivalent rate, CE. A certainty equivalent rate is the annual riskless rate that would provide the maximized expected utility $\mathsf{E}\left[U\left(W_{T+K}(\omega^*)\right)\right]$. It is given by:

$$\mathrm{CE} = \left\{(1-\gamma)\mathsf{E}\left[U\left(W_{T+K}(\omega^*)\right)\right]\right\}^{\frac{1}{H(1-\gamma)}} - 1, \qquad (18)$$

where H is the length of horizon in years, i.e., $H = \frac{K}{12}$ and $H = \frac{K}{4}$ for monthly and quarterly observations, respectively. The lower bound on CE is the annual risk free rate of return prevailing over the investment horizon.

The overall pattern of asset allocations, as displayed in Table 9, is consistent with return predictability. The optimal portfolio choices exhibit high sensitivity to the current values of predictive variables. In particular, centering those values around the sample means rather than the actual end-of-sample realizations modifies the compositions of risky assets in the optimal portfolio. For example, focusing on $\gamma = 3$ and a one-year investment horizon, the invested wealth in the small size, high book-to-market portfolio increases from 16.84% to 44.48%, whereas the wealth invested in the big size, high book-to-market portfolio decreases from 82.16% to 54.54%. The investment in the other equity portfolios remain zero.

Focusing on the actual end-of-sample realizations to explore the attractiveness of investment opportunities, we show that investors tend to allocate more to equities the longer their horizon. For example, with $\gamma = 6$ the total allocation to equities is 52.23% and 54.38% for horizons of one and ten years, respectively. In the same vein, investment opportunities, as summarized by the certainty equivalent measure, are perceived more attractive with longer horizons. For example, annual certainty equivalent rates corresponding to horizons of one and ten years are 7.22% and 7.69%, respectively.

It should be noted that the horizon effect found here is not as robust as the one documented by Barberis (2000). Focusing on the dividend yield as a single predictor, Barberis (2000) shows that investors allocate substantially more to stocks the longer their horizon. In our analysis, which explicitly accounts for model uncertainty, the increase in allocation to equities for longer horizons is fairly modest and completely disappears when the current values of the predictive variables are equal to their sample means. The disappearance of the horizon effect is consistent with Heaton and Lucas (2000) and Ameriks and Zeldes (2000) who show that older people (probably shorter horizon investors) could hold more in stocks than younger cohorts. Interestingly, Ameriks and Zeldes (2000) also show that almost half of their sample members made no active changes to their portfolio allocation, i.e., those are buy-and-hold investors similar to the one examined in our study.

V Conclusion

We use Bayesian model averaging to investigate the sample evidence about return predictability in the presence of model uncertainty. The analysis shows that such uncertainly is more important than the within-model parameter uncertainly for monthly observations, but the reverse is true for quarterly observations. Incorporating model uncertainty undermines the apparent predictive power of several economic variables. However, both in-sample and out-of-sample evidence based on posterior and predictive analysis support predictability.

We also show that the out-of-sample performance of Bayesian model averaging is superior to that of the traditional model selection criteria studied by Bossaerts and Hillion (1999).

Several economic variables are found useful predictors of future returns on portfolios sorted on size and book-to-market. Such variables include the difference between lagged returns on long-term and short-term government bonds, earnings yield, and Treasury-bill rate. Interestingly, the trend-deviation-in-wealth appears strong in forecasting future returns when the set of predictive variables is restricted to that studied by Lettau and Ludvigson (2000). However, its predictive power is somewhat undermined when the information set is expanded to include several other variables. Last, we show that small high book-to-market stocks are more predictable than big low-book-to-market stocks.

Directions for future research include the implementation of our methodology to examine predictability of returns on fixed income securities, of forward premiums, and of economic growth. Our approach gives interesting directions for examining non-nested models as well. For example, one can compute posterior probabilities for GARCH and stochastic volatility models in order to select the optimal model or, instead, to average across these models. The uncertainty about the true volatility model is especially relevant in pricing derivative securities, but it also can provide powerful insight for short-term asset allocation decisions.

The study derives portfolio choice in a simplified environment, focusing on descriptive implications. The normative implications of model uncertainty for asset allocation decisions merit further research. In particular, can model uncertainty induce hedging demands for risky assets? Finally, in a general equilibrium setting, investors who face model uncertainty will require an extra premium for holding equities. Estimating the equity premium in the presence of model uncertainty is of great interest, especially when the current estimates appear too large to be reconciled with the perceived uncertainty about stock returns.

A Proof of Proposition 1

First note that all the various quantities based on the hypothetical sample, denoted by the subscript 0, must be expressed in terms of quantities observed from the actual sample. In particular (the model-specific-subscript is suppressed for notational clarity):

$$\frac{1}{T_0}(X_0'X_0) = \frac{1}{T}(X'X) = \begin{bmatrix} 1 & \bar{z}' \\ \bar{z} & \bar{z}\bar{z}' + \hat{V}_z \end{bmatrix}, \tag{A.1}$$

$$\begin{aligned} X_0'R_0 &= (X_0'X_0)B_0, \\ &= \frac{T_0}{T}(X'X)\begin{bmatrix} \bar{r}' \\ 0 \end{bmatrix}, \\ &= T_0 \begin{bmatrix} \bar{r}' \\ \bar{z}\bar{r}' \end{bmatrix}. \end{aligned} \tag{A.2}$$

The joint posterior distribution of B and Σ based on the hypothetical sample is the prior distribution for those parameters based on the actual sample

$$P(B,\Sigma) \propto |\Sigma|^{-\frac{T_0}{2}} \exp\left(-\frac{1}{2}\text{tr}\left[S_0 + (B-B_0)'X_0'X_0(B-B_0)\right]\Sigma^{-1}\right), \tag{A.3}$$

where

$$\begin{aligned} S_0 &= (R_0 - X_0B_0)'(R_0 - X_0B_0), \\ &= (R_0 - \iota_{T_0}\bar{r})'(R_0 - \iota_{T_0}\bar{r}), \\ &= T_0\hat{V}_r, \end{aligned} \tag{A.4}$$

and ι_{T_0} is a $T_0 \times 1$ vector of ones. Standard results (e.g., Zellner (1971)) imply that Σ obeys the inverted Wishart distribution with a parameter matrix S_0 and $T_0 - N - 1$ degrees of freedom. Conditional on Σ, the vector $b = \text{vec}(B)$ is multivariate normally distributed with mean $b_0 = \text{vec}(B_0)$ and variance $\Sigma \otimes (X'X)^{-1}$. The informative priors for B and Σ can be expresses as

$$P\big(b|\Sigma\big) = (2\pi)^{-\frac{N(m+1)}{2}} |\Sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\big(b-b_0\big)'\big[\Sigma^{-1} \otimes X_0'X_0\big]\big(b-b_0\big)\right), \quad \text{(A.5)}$$
$$P\big(\Sigma\big) = \psi_0 |S_0|^{\frac{T_0-N-1}{2}} |\Sigma|^{-\frac{T_0}{2}} \exp\left(-\frac{1}{2}\text{tr}\big[S_0\Sigma^{-1}\big]\right),$$

where

$$\psi_0 = \left(2^{\frac{(T_0-N-1)N}{2}} \pi^{\frac{N(N-1)}{4}} \prod_{i=1}^{N} \Gamma\left[\frac{T_0-N-i}{2}\right]\right)^{-1}. \quad \text{(A.6)}$$

The normalization constants must be included while computing the marginal likelihood and are therefore displayed above.

The likelihood function (the one that integrates to unity) of normally distributed data constituting the actual sample obeys the form:

$$P(D|B,\Sigma) = (2\pi)^{-\frac{TN}{2}} |\Sigma|^{-\frac{T}{2}} \exp\left(-\frac{1}{2}\text{tr}\left[S + (B-\hat{B})'X'X(B-\hat{B})\right]\Sigma^{-1}\right), \quad \text{(A.7)}$$

where

$$S = (R - X\hat{B})'(R - X\hat{B}), \quad \text{(A.8)}$$
$$\hat{B} = \big(X'X\big)^{-1} X'R. \quad \text{(A.9)}$$

Combining the likelihood (A.7) and the prior (A.3) and completing the square on b yield

$$P\big(b|\Sigma,D\big) = (2\pi)^{-\frac{N(m+1)}{2}} |\Sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\big(b-\tilde{b}\big)'\big[\Sigma^{-1} \otimes (X_0'X_0 + X'X)\big]\big(b-\tilde{b}\big)\right), \quad \text{(A.10)}$$
$$P\big(\Sigma|D\big) = \psi |\tilde{S}|^{\frac{\nu}{2}} |\Sigma|^{-\frac{\nu+N+1}{2}} \exp\left(-\frac{1}{2}\text{tr}\big[\tilde{S}\Sigma^{-1}\big]\right),$$

where

$$\begin{aligned}
\tilde{b} &= \text{vec}(\tilde{B}), \\
\tilde{B} &= (X_0'X_0 + X'X)^{-1}(X_0'R_0 + X'R), \\
\tilde{S} &= R'R + S_0 + R_0'X_0(X_0'X_0)^{-1}X_0'R_0 - \tilde{B}'(X_0'X_0 + X'X)\tilde{B}, \\
\psi &= \left(2^{\frac{\nu N}{2}} \pi^{\frac{N(N-1)}{4}} \prod_{i=1}^{N} \Gamma\left[\frac{\nu+1-i}{2}\right]\right)^{-1}, \\
\nu &= T_0 + T - N - 1.
\end{aligned}$$

The marginal likelihood is the product of the prior and likelihood divided by the posterior, all of which are evaluated at an arbitrary point in the parameter space B^* and Σ^*

$$P(D|\mathcal{M}_j) = \frac{P(D|\Sigma^*, B^*, \mathcal{M}_j)\, P(\Sigma^*, B^*|\mathcal{M}_j)}{P(\Sigma^*, B^*|D, \mathcal{M}_j)}. \tag{A.11}$$

Computing the log marginal likelihood is straightforward: take logs from both sides of (A.11) and replace the prior, likelihood, and posterior densities in (A.5), (A.7), and (A.10), respectively, by their corresponding normalization constants.

B The Joint Posterior Distribution of Φ and Ψ

To solve for the posterior distribution of Φ and Ψ, we follow Kandel and Stambaugh (1996) and make the additional assumption that the prior sample produces the same values as the actual counterpart for the statistics corresponding to ρ and $\tilde{z}$, where

$$\rho = \frac{1}{T}\sum_{t=0}^{T-1}(z_t - \bar{z})(z_{t+1} - \tilde{z})' \tag{B.1}$$

is the matrix of autocorrelation and cross autocorrelation of m predetermined variables and $\tilde{z} = \frac{1}{T}\sum_{t=1}^{T} z_t$, results in an informative joint posterior distribution of Φ and Ψ:

$$\text{vec}(\Phi)|\Psi \sim N\left(\text{vec}(\Phi_0), \Psi \otimes (X_0'X_0)^{-1}\right), \tag{B.2}$$

$$\Psi \sim IW\left(\Psi_0, T_0 - (N+m) - 1\right),$$

where

$$\begin{aligned}
\Phi_0 &= (X_0'X_0)^{-1}(X_0'Y_0), \\
&= \left[B_0, (X_0'X_0)^{-1}(X_0'Z_0)\right], \\
X_0'Z_0 &= T_0 \begin{bmatrix} \tilde{z}' \\ \rho + \bar{z}\tilde{z}' \end{bmatrix}, \\
\Psi_0 &\approx T_0 \begin{bmatrix} \hat{V}_r & V \\ V & \hat{V}_z + \bar{z}\bar{z}' - \frac{1}{T_0} Z_0'X_0(X_0'X_0)^{-1}X_0'Z_0 \end{bmatrix}.
\end{aligned}$$

The approximation becomes equality if the first and last observations of the predictive variables are equal. The off-diagonal matrix V is assumed zero, an innocuous assumption. Combining the joint prior distribution in (B.2) with normally distributed data constituting the primary sample, the posterior distributions for $\phi = \text{vec}(\Phi)$ and Ψ are obtained as

$$\phi|\Psi, D \sim N\left(\tilde{\phi}, \Psi \otimes (X_0'X_0 + X'X)^{-1}\right), \tag{B.3}$$

$$\Psi|D \sim IW\left(\tilde{\Psi}, T + T_0 - (N+m) - 1\right),$$

where

$$\begin{aligned}
\tilde{\phi} &= \text{vec}(\tilde{\Phi}), \\
\tilde{\Phi} &= (X_0'X_0 + X'X)^{-1}(X_0'Y_0 + X'Y), \\
\tilde{\Psi} &= Y'Y + \Psi_0 + Y_0'X_0(X_0'X_0)^{-1}X_0'Y_0 - \tilde{\Phi}'(X_0'X_0 + X'X)\tilde{\Phi}.
\end{aligned}$$

C Proof of Proposition 2

Partitioning equation (11) yields

$$(r_t', z_t') = (1, z_{t-1}') \begin{bmatrix} b' & a' \\ C' & A \end{bmatrix} + \begin{pmatrix} \epsilon_t \\ e_t \end{pmatrix}', \tag{C.1}$$

where

$$\begin{pmatrix} \epsilon_t \\ e_t \end{pmatrix} \sim N\left(0, \begin{bmatrix} \Sigma & \Lambda \\ \Lambda' & \Phi \end{bmatrix}\right). \tag{C.2}$$

It follows from equation (C.1) that:

$$r_{T+1} = b + Cz_T + \epsilon_{T+1}, \tag{C.3}$$

$$z_{T+1} = a + A'z_T + e_{T+1}. \tag{C.4}$$

The cumulative excess return over the investment horizon is computed as

$$\begin{aligned} R_{T+K} &= \sum_{k=1}^{K} r_{t+k}, \\ &= Kb + C\left(\sum_{j=1}^{K} z_{T+j-1}\right) + \sum_{j=1}^{K} \epsilon_{T+j}, \end{aligned} \tag{C.5}$$

where z_{T+j} is obtained by iterating over equation (C.4). In particular,

$$z_{T+J} = [(A')^J - I_m](A' - I_m)^{-1}a + (A')^J z_T + \sum_{j=1}^{J} (A')^{J-j} e_{T+j}. \tag{C.6}$$

Substituting equation (C.6) into equation (C.5) for $J = 1, \dots, K-1$ yields:

$$\begin{aligned} R_{T+K} &= Kb + C\left[A'\left((A')^{K-1} - I_m\right)(A' - I_m)^{-1} - (K-1)I_m\right](A' - I_m)^{-1}a \\ &+ C\left((A')^K - I_m\right)(A' - I_m)^{-1} z_T + \sum_{j=2}^{K}\sum_{i=1}^{j-1} C(A')^{j-i-1} e_{T+i} + \sum_{j=1}^{K} \epsilon_{T+j}, \end{aligned}$$

for $K \geq 2$. The results follow immediately.

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Table 1
Descriptive Statistics of Predictive Variables and Monthly Stock Returns

The table shows descriptive statistics based on the actual sample spanning 549 months from April 1953 to December 1998 for monthly continously compounded returns on six equity portfolios and 13 predictors. The portfolios are identified by a combination of two letters designating increasing values of size (S,B) and book-to-market (L,M,H). The 13 predictors are: dividend yield on the value weighted NYSE index (Div); book-to-market (BM) on the Standard & Poor's Industrials; earnings yield on the Standard & Poor's Composite index (EY); the one-year momentum portfolio (WML); the difference in annualized yields of Moody's Baa and Aaa rated bonds (Def); the monthly rate of a three-month Treasury bill (Tbill); excess return on the value weighted index (Ret); the difference between the return on long-term corporate bonds and the return on long-term government bond (DEF); the difference between the monthly return on long-term government bond and the one month Treasury bill rate (TERM); the inflation rate (Inf); size premium (SMB); value premium (HML); and the difference in annualized yield of ten-year and one-year Treasury bills (Term). Std.Dev. denotes the standard deviation. The parameter ρ_t is the sample autocorrelation at lag t months.

Statistic:	Mean	Std.Dev.	ρ_1	ρ_3	ρ_6	ρ_{12}	ρ_{60}
Predictive Variables:							
Div	0.0362	0.0091	0.9828	0.9478	0.8847	0.7620	0.3276
BM	0.5048	0.1735	0.9889	0.9674	0.9304	0.8572	0.4912
EY	0.8531	0.2936	0.9929	0.9679	0.9162	0.7981	0.3638
WML	0.0097	0.0357	-0.0377	-0.1016	0.0706	0.2347	0.2293
Def	0.9476	0.4385	0.9738	0.9106	0.8360	0.6941	0.3859
Tbill	0.0044	0.0024	0.9565	0.9113	0.8638	0.7818	0.4258
Ret	0.0063	0.0423	0.0655	0.0041	-0.0650	0.0312	-0.0504
DEF	0.0003	0.0115	-0.1881	-0.0493	-0.0434	0.0054	0.0088
TERM	0.0011	0.0263	0.0662	-0.1037	0.0452	-0.0107	-0.0242
Inf	0.3330	0.3334	0.5541	0.4755	0.4416	0.5152	0.2929
SMB	0.0009	0.0262	0.1659	-0.0134	0.0708	0.1871	0.0305
HML	0.0039	0.0244	0.1483	-0.0077	0.0430	0.1013	0.0063
Term	0.7195	0.9908	0.9589	0.8368	0.7033	0.5071	0.0217
Equity Portfolios:							
SL	0.0098	0.0614	0.1722	-0.0242	-0.0237	0.0085	-0.0401
SM	0.0130	0.0501	0.1854	-0.0122	-0.0010	0.0694	0.0034
SH	0.0149	0.0509	0.1795	-0.0275	-0.0132	0.1272	0.0482
BL	0.0108	0.0451	0.0571	0.0013	-0.0665	0.0535	-0.0770
BM	0.0110	0.0399	0.0129	0.0127	-0.0660	0.0057	-0.0262
BH	0.0132	0.0434	0.0443	0.0244	-0.0214	0.0544	0.0026

Table 2
Multiple Regressions of Monthly Excess Continously Compounded Returns on Predictive Variables

The table displays OLS estimates based on six multiple regressions of excess continously compounded returns on a constant intercept and 14 predictive variables. Reported are slope coefficients (top figures) and ratios obtained by dividing the slopes by their corresponding standard deviations (bottom figures). Excess returns are on six portfolios formed as the intersection of two size (S,B) and three book-to-market (L,M,H) groups. The set of predictors includes: dividend yield (Div); book-to-market (BM); earnings yield (EY); the one-year momentum portfolio (WML); default risk spread (Def); the monthly rate of a three-month Treasury bill (Tbill); excess return on the value weighted index (Ret); default risk premium (DEF); term-structure premium (TERM); the inflation rate (Inf); size premium (SMB); value premium (HML); and the term-structure spread (Term).

Portfolio:	SL	SM	SH	BL	BM	BH
Div	-0.0129	-0.0105	-0.0076	0.0018	-0.0007	-0.0027
	-0.9936	-1.0145	-0.7334	0.1903	-0.0880	-0.2999
BM	-0.0018	-0.1591	-0.4942	0.3266	0.0759	-0.1280
	-0.0019	-0.2096	-0.6525	0.4655	0.1234	-0.1935
EY	-0.0655	-0.0347	-0.0134	-0.0651	-0.0466	-0.0114
	-1.6370	-1.0873	-0.4214	-2.2042	-1.8025	-0.4085
WML	0.0782	0.0568	0.0541	0.0441	0.0450	0.0300
	2.5494	2.3216	2.2125	1.9495	2.2683	1.4055
Def	-0.0217	-0.0228	-0.0529	-0.0093	0.0311	-0.0272
	-0.2689	-0.3541	-0.8242	-0.1562	0.5970	-0.4852
Tbill	0.0160	0.0096	0.0061	0.0124	0.0113	0.0076
	1.4745	1.1137	0.7062	1.5503	1.6157	1.0097
Ret	-6.7958	-3.9722	-3.1236	-4.0756	-4.6298	-2.7374
	-3.1139	-2.2808	-1.7974	-2.5313	-3.2827	-1.8032
DEF	0.0191	0.0443	0.0751	-0.0795	-0.1034	-0.0566
	0.2460	0.7173	1.2177	-1.3906	-2.0659	-1.0504
TERM	0.4310	0.3254	0.2454	0.4139	0.3534	0.2117
	1.6003	1.5140	1.1442	2.0831	2.0304	1.1300
Jan	0.3160	0.3421	0.2948	0.2619	0.2973	0.2170
	2.5537	3.4649	2.9923	2.8686	3.7177	2.5207
Inf	0.0238	0.0289	0.0430	-0.0044	0.0079	0.0268
	2.4281	3.6917	5.5041	-0.6045	1.2495	3.9225
SMB	-0.0138	-0.0124	-0.0121	-0.0182	-0.0090	-0.0124
	-1.3518	-1.5155	-1.4885	-2.4023	-1.3552	-1.7372
HML	0.1932	0.1285	0.1109	0.0720	0.0770	0.0264
	1.7380	1.4489	1.2525	0.8776	1.0713	0.3409
Term	-0.1125	-0.0014	0.0860	-0.0790	0.0119	0.0249
	-0.9558	-0.0151	0.9177	-0.9097	0.1559	0.3037

Table 3

Posterior Probabilities of Forecasting Models Based on a Prior Sample Weighted against Predictability

The top figures denote the highest-posterior probability compositions represented by a combination of zeros and ones designating exclusions and inclusions of predictive variables, respectively. The bottom figures display cumulative posterior probabilities computed as $\mathcal{A}'\mathcal{P}$, where $\mathcal{A}$ is a $2^{14} \times 14$ matrix representing all forecasting models by their unique combinations of zeros and ones and $\mathcal{P}$ is a $2^{14} \times 1$ vector including posterior probabilities for all models. The stock universe comprises six portfolios identified by two letters designating increasing values of size (S, B) and book-to-market (L, M, H). Following are the predictors spanning the information set: dividend yield (Div); book-to-market (BM); earnings yield (EY); the momentum portfolio (WML); the difference in annualized yields of Moody's Baa and Aaa rated bonds (Def); the monthly rate of a three-month Treasury bill (Tbill); excess return on the value weighted index (Ret); the difference between the return on long-term corporate bonds and the return on long-term government bond (DEF); the difference between the monthly return on long-term government bond and the one month Treasury bill rate (TERM); the inflation rate (Inf); size premium (SMB); value premium (HML); and the difference in annualized yield of ten-year and one-year Treasury bills (Term). Figures displayed below are computed when investors perceive the events of predictability versus no predictability as equally likely prior to encountering a hypothetical sample weighted against predictability.

	Predictive Variables													
	Div	BM	EY	WML	Def	Tbill	Ret	DEF	TERM	Jan	Inf	SMB	HML	Term
Portfolio:														
SL	0	1	1	0	1	1	0	1	1	1	0	1	0	0
	0.45	0.53	0.80	0.38	0.58	0.86	0.52	0.51	0.72	0.76	0.57	0.62	0.45	0.44
SM	0	0	1	0	0	1	0	1	1	1	0	1	0	1
	0.43	0.41	0.69	0.36	0.48	0.67	0.54	0.50	0.91	0.95	0.54	0.54	0.33	0.57
SH	0	0	1	0	0	0	1	0	1	1	1	0	0	1
	0.40	0.37	0.60	0.39	0.40	0.55	0.61	0.40	0.81	1.00	0.49	0.48	0.34	0.62
BL	0	1	1	0	1	1	0	1	1	0	1	0	0	0
	0.53	0.61	0.65	0.43	0.58	0.71	0.45	0.64	0.78	0.44	0.81	0.47	0.47	0.49
BM	0	1	1	0	1	1	1	1	1	0	1	0	0	0
	0.50	0.54	0.69	0.41	0.61	0.87	0.53	0.62	0.92	0.54	0.57	0.45	0.41	0.52
BH	0	0	1	0	0	0	0	0	1	1	1	0	0	1
	0.46	0.44	0.51	0.40	0.51	0.61	0.39	0.45	0.73	0.98	0.64	0.39	0.40	0.66

Table 4
Slope Coefficients in the Weighted Model and their t-Ratios

The top figures denote posterior means of slope coefficients obtained by averaging slope estimates across models:

$$\mathsf{E}(B|D) = \sum_{j=1}^{2^M} P(\mathcal{M}_j|D)\,\tilde{B}_j.$$

The middle and bottom figures denote t-ratios unadjusted and adjusted to account for model uncertainty, respectively. In particular, the former is obtained by dividing the posterior mean of each of the slope coefficients by its posterior standard error corresponding to the first variance component in the following equation:

$$\mathrm{Var}(B|D) = \sum_{j=1}^{2^M} P(\mathcal{M}_j|D) \left\{ \frac{T\tilde{S}_j(X_j'X_j)^{-1}}{T_j^*(T_j^*-4)} + \left[\tilde{B}_j - \mathsf{E}(B|D)\right]\left[\tilde{B}_j - \mathsf{E}(B|D)\right]' \right\}.$$

The latter divides the posterior mean by the posterior standard error corresponding to the overall variance, including model uncertainty that summarizes the dispersion in slopes across models. The statistics are computed separately for each of six equity portfolios formed as the intersection of two size (S, B) and three book-to-market (L, M, H) groups. Following are the predictors spanning the information set: dividend yield (Div); book-to-market (BM); earnings yield (EY); the momentum portfolio (WML); the difference in annualized yields of Moody's Baa and Aaa rated bonds (Def); the monthly rate of a three-month Treasury bill (Tbill); excess return on the value weighted index (Ret); the difference between the return on long-term corporate bonds and the return on long-term government bond (DEF); the difference between the monthly return on long-term government bond and the one month Treasury bill rate (TERM); the inflation rate (Inf); size premium (SMB); value premium (HML); and the difference in annualized yield of ten-year and one-year Treasury bills (Term). Figures displayed below are computed when investors perceive the events of predictability versus no predictability as equally likely prior to encountering a hypothetical sample weighted against predictability.

	Predictive Variables													
	Div	BM	EY	WML	Def	Tbill	Ret	DEF	TERM	Jan	Inf	SMB	HML	Term
Portfolio:														
SL	0.06	-0.01	0.03	-0.01	0.01	-2.57	0.03	0.11	0.10	0.01	-0.01	0.07	-0.03	0.00
	0.15	-0.76	1.76	-0.17	0.98	-2.19	0.84	0.79	1.50	1.61	-1.00	1.15	-0.56	0.41
	0.12	-0.56	1.22	-0.16	0.69	-1.48	0.61	0.59	1.05	1.18	-0.72	0.84	-0.47	0.34
SM	0.06	0.00	0.01	-0.01	0.00	-1.25	0.03	0.09	0.15	0.02	0.00	0.05	0.00	0.00
	0.21	-0.28	1.38	-0.30	0.80	-1.54	1.03	0.85	2.44	2.67	-1.03	1.00	-0.09	1.11
	0.16	-0.22	0.94	-0.27	0.54	-0.96	0.71	0.62	1.80	2.23	-0.72	0.72	-0.09	0.75
SH	0.03	0.00	0.01	-0.02	0.00	-0.86	0.04	0.06	0.12	0.02	0.00	0.04	0.01	0.00
	0.10	0.13	1.15	-0.57	0.60	-1.20	1.37	0.60	2.07	3.99	-0.94	0.89	0.36	1.36
	0.08	0.10	0.78	-0.46	0.41	-0.75	0.88	0.45	1.40	3.74	-0.65	0.63	0.31	0.89
BL	0.15	-0.01	0.01	0.00	0.00	-1.10	-0.01	0.12	0.09	0.00	-0.01	0.02	-0.02	0.00
	0.54	-0.96	1.07	-0.18	0.85	-1.40	-0.21	1.08	1.62	-0.23	-1.79	0.43	-0.43	0.45
	0.42	-0.70	0.77	-0.17	0.63	-0.94	-0.19	0.80	1.18	-0.22	-1.30	0.39	-0.38	0.38
BM	0.10	-0.01	0.01	0.00	0.00	-1.64	-0.02	0.10	0.12	0.00	0.00	0.01	0.00	0.00
	0.42	-0.71	1.23	0.20	1.01	-2.16	-0.73	1.07	2.36	0.77	-0.90	0.42	0.12	0.68
	0.33	-0.53	0.89	0.20	0.72	-1.50	-0.58	0.80	1.84	0.62	-0.68	0.37	0.11	0.52
BH	0.08	0.00	0.00	-0.01	0.00	-0.77	0.00	0.04	0.07	0.01	0.00	0.01	0.01	0.00
	0.34	0.14	0.58	-0.29	0.74	-1.16	-0.16	0.45	1.48	2.87	-1.22	0.18	0.24	1.29
	0.30	0.12	0.48	-0.27	0.53	-0.78	-0.15	0.38	1.08	2.54	-0.87	0.17	0.23	0.90

Table 5

Posterior Odds in Favor of Predictability Based on Various Hypothetical Prior Samples

The table exhibits posterior odds in favor of predictability, or against the iid model, for various values of the hypothetical sample size, ranging from 20 to 1280 years per parameter. The prior-odds ratio in favor of predictability is set equal to unity. The posterior-odds ratio is computed by dividing the sum of posterior probabilities assigned to models that retain predictors by the posterior probability of the iid model. The stock universe comprises six portfolios identified by two letters designating increasing values of size (S, B) and book-to-market (L, M, H). Following are the predictors constituting the information set: dividend yield (Div); book-to-market (BM); earnings yield (EY); the momentum portfolio (WML); the difference in annualized yields of Moody's Baa and Aaa rated bonds (Def); the monthly rate of a three-month Treasury bill (Tbill); excess return on the value weighted index (Ret); the difference between the return on long-term corporate bonds and the return on long-term government bond (DEF); the difference between the monthly return on long-term government bond and the one month Treasury bill rate (TERM); the inflation rate (Inf); size premium (SMB); value premium (HML); and the difference in annualized yield of ten-year and one-year Treasury bills (Term).

Number of years per parameter	20	40	80	160	320	640	1280
Portfolio:							
			Posterior-Odds Ratios				
SL	39.07	7.37	2.84	1.71	1.31	1.15	1.07
SM	113.72	12.82	3.76	1.96	1.40	1.19	1.09
SH	295.49	19.83	4.54	2.13	1.46	1.21	1.10
BL	8.85	3.24	1.84	1.37	1.17	1.08	1.04
BM	14.22	4.14	2.09	1.45	1.21	1.10	1.05
BH	23.47	5.35	2.37	1.55	1.24	1.12	1.06

Table 6

Posterior Probabilities of Forecasting Models Using Quarterly Observations

The table exhibits cumulative posterior probabilities for fourteen predictive variables computed as $\mathcal{A}'\mathcal{P}$, where $\mathcal{A}$ is a $2^{14} \times 14$ matrix representing all forecasting models by their unique combinations of zeros and ones designating exclusions and inclusions of predictors, respectively, and $\mathcal{P}$ is a $2^{14} \times 1$ vector including posterior probabilities for all models. Figures displayed below are computed when investors perceive the events of predictability versus no predictability as equally likely prior to encountering a hypothetical no-predictability informative sample taking values equivalent to 17 observations per parameter. The asset universe comprises six equity portfolios identified by two letters designating increasing values of size (S, B) and book-to-market (L, M, H). Following are the predictors constituting the information set: dividend yield (Div); book-to-market (BM); earnings yield (EY); momentum (WML); default risk spread (Def); the three-month rate of a three-month Treasury bill (Tbill); a quarterly excess return on the value weighted index (Ret); default risk premium (DEF); term structure premium (TERM); trend deviation in wealth (TDW); the three-month inflation rate (Inf); size premium (SMB); value premium (HML); and the term spread (Term).

	Predictive Variables													
	Div	BM	EY	WML	Def	Tbill	Ret	DEF	TERM	TDW	Inf	SMB	HML	Term
Portfolio:														
	Prior Sample Size Equivalent to 17 Observations per Parameter													
SL	0.49	0.53	0.75	0.49	0.49	0.85	0.43	0.72	0.81	0.63	0.43	0.62	0.42	0.43
SM	0.49	0.48	0.67	0.48	0.51	0.73	0.45	0.69	0.76	0.65	0.45	0.70	0.42	0.47
SH	0.49	0.47	0.65	0.53	0.51	0.70	0.53	0.65	0.73	0.54	0.46	0.69	0.43	0.51
BL	0.55	0.60	0.63	0.50	0.52	0.77	0.45	0.66	0.73	0.54	0.49	0.83	0.43	0.46
BM	0.52	0.55	0.70	0.44	0.60	0.87	0.45	0.51	0.61	0.58	0.47	0.80	0.42	0.48
BH	0.51	0.47	0.57	0.44	0.54	0.68	0.62	0.55	0.59	0.53	0.55	0.85	0.43	0.51

Table 7

Slope Coefficients in the Weighted Model and their t-Ratios: The case of Quarterly Observations

The top and bottom figures denote t-ratios unadjusted and adjusted to account for model uncertainty, respectively. In particular, the former is obtained by dividing the posterior mean of each of the slope coefficients obtained by averaging slope estimates across models by the posterior standard error corresponding to the first variance component in the following equation:

$$\mathrm{Var}(B|D) = \sum_{j=1}^{2^M} P(\mathcal{M}_j|D) \left\{ \frac{T\tilde{S}_j(X_j'X_j)^{-1}}{T_j^*(T_j^*-4)} + \left[\tilde{B}_j - \mathsf{E}(B|D)\right]\left[\tilde{B}_j - \mathsf{E}(B|D)\right]' \right\}.$$

The latter divides the posterior mean by the posterior standard error corresponding to the overall variance, including model uncertainty that summarizes the dispersion in slopes across models. The hypothetical no-predictability informative sample takes values equivalent to 17 observations per parameter. The statistics are computed separately for each of six equity portfolios formed as the intersection of two size (S, B) and three book-to-market (L, M, H) groups. Following are the predictors constituting the information set: dividend yield (Div); book-to-market (BM); earnings yield (EY); momentum (WML); default risk spread (Def); the three-month rate of a three-month Treasury bill (Tbill); a quarterly excess return on the value weighted index (Ret); default risk premium (DEF); term structure premium (TERM); trend deviation in wealth (TDW); the three-month inflation rate (Inf); size premium (SMB); value premium (HML); and the term spread (Term). Figures displayed below are computed when investors perceive the events of predictability versus no predictability as equally likely prior to encountering a hypothetical sample weighted against predictability.

	Predictive Variables													
	Div	BM	EY	WML	Def	Tbill	Ret	DEF	TERM	TDW	Inf	SMB	HML	Term
Portfolio:														
	Prior Sample Size Equivalent to 17 Observations per Parameter													
SL	0.30	-0.62	1.44	0.57	0.45	-1.96	-0.11	1.41	1.80	1.09	-0.08	-1.02	-0.17	-0.06
	0.25	-0.47	1.03	0.47	0.35	-1.41	-0.10	1.02	1.30	0.83	-0.07	-0.78	-0.16	-0.05
SM	0.30	-0.33	1.15	0.52	0.58	-1.48	0.30	1.32	1.61	1.16	-0.31	-1.30	-0.25	0.39
	0.25	-0.27	0.84	0.44	0.43	-1.02	0.25	0.95	1.13	0.87	-0.27	-0.97	-0.23	0.32
SH	0.19	-0.08	1.04	0.69	0.55	-1.34	0.71	1.15	1.47	0.75	-0.32	-1.26	-0.13	0.57
	0.16	-0.07	0.79	0.55	0.41	-0.94	0.54	0.84	1.03	0.60	-0.28	-0.93	-0.13	0.46
BL	0.63	-0.89	0.98	-0.48	0.51	-1.59	0.08	1.13	1.42	0.70	-0.47	-1.74	-0.08	0.09
	0.49	-0.65	0.72	-0.41	0.40	-1.11	0.07	0.84	1.03	0.58	-0.39	-1.34	-0.08	0.08
BM	0.46	-0.71	1.24	-0.22	0.91	-2.10	0.28	0.65	1.01	0.88	-0.45	-1.65	-0.18	0.40
	0.36	-0.53	0.90	-0.20	0.66	-1.47	0.24	0.52	0.75	0.69	-0.38	-1.25	-0.17	0.33
BH	0.35	-0.01	0.71	-0.12	0.71	-1.26	1.03	0.77	0.92	0.70	-0.74	-1.89	0.16	0.56
	0.31	-0.01	0.57	-0.11	0.52	-0.88	0.76	0.60	0.69	0.57	-0.58	-1.44	0.15	0.46

Table 8

Bayesian Model Averaging: Out-of-Sample Performance

The table reports the sum of squared forecast errors (SSE), the sum of forecast errors (SFE), and the standard deviation of forecast errors (SDE) for seven specifications. These are the weighted forecasting model (WFM), the all-inclusive model (ALL), the iid model (IID), and the models selected by the criteria AIC, SIC, FIC, and PIC. Out-of-sample forecast errors are computed using the following algorithm. Based upon the initial $t = \frac{T}{3}$ sample observations, we compute posterior probabilities for all 2^M compositions and select four models based on the aforementioned criteria. Next, we project the time $t+1$ excess return for each of the seven specifications and retain the corresponding forecast errors. The excess return and predictive variables realized at time $t+1$ are then added to the data set to revise both the model selection and posterior probability computation and to project the time $t+2$ excess return. Following these steps, we obtain $2 \times \frac{T}{3}$ out-of-sample forecast errors for each specification.

	WFM	ALL	IID	AIC	SIC	FIC	PIC
Statistic:							
			Monthly Observations				
SSE	0.7793	0.8165	0.7886	0.8010	0.8270	0.8187	0.8195
SFE	-0.0224	0.1671	-0.3488	0.4572	0.4641	1.0360	0.7383
SDE	0.0462	0.0473	0.0465	0.0468	0.0476	0.0473	0.0473
			Quarterly Observations				
SSE	0.9314	1.0076	0.9492	0.9966	1.1073	1.0155	1.0173
SFE	0.1760	0.8858	-0.3785	1.1395	1.3814	0.8844	0.8258
SDE	0.0877	0.0910	0.0885	0.0903	0.0950	0.0913	0.0914

Table 9
Asset Allocations Based on the Weighted Model

The table exhibits allocations to six size book-to-market portfolios as percentages of the total invested wealth for both monthly (Panel A) and quarterly (Panel B) observations when the recent values of the predictive variables (z_T) are equal to the actual realizations, as documented at the end-of-sample period, and to the sample means. Asset allocations are derived for investment horizons ranging from one to ten years and relative risk-aversion coefficient (γ) equal to three, six, and nine. Also reported are total allocations to equities (Total) and a certainty equivalent measure (CE) defined as the annual riskless rate that would provide expected utility equal to the one obtained based on the optimal allocations displayed below.

Panel A: The Case of Monthly Observations

Horizon	SL	SM	SH	BL	BM	BH	Total	CE	SL	SM	SH	BL	BM	BH	Total	CE
	z_T=End-of-Sample Realizations								z_T=Sample Means							
								γ=3								
1	0.00	0.00	53.67	0.00	0.00	45.33	99.00	11.69	0.00	0.00	77.29	0.00	0.00	21.71	99.00	13.18
2	0.00	0.00	55.89	0.00	0.00	43.11	99.00	11.73	0.00	0.00	75.62	0.00	0.00	23.38	99.00	13.02
3	0.00	0.00	58.04	0.00	0.00	40.96	99.00	11.86	0.00	0.00	76.23	0.00	0.00	22.77	99.00	12.96
4	0.00	0.00	59.73	0.00	0.00	39.27	99.00	12.04	0.00	0.00	75.57	0.00	0.00	23.43	99.00	13.08
5	0.00	0.00	64.35	0.00	0.00	34.65	99.00	12.18	0.00	0.00	75.61	0.00	0.00	23.39	99.00	12.98
6	0.00	0.00	63.41	0.00	0.00	35.59	99.00	12.34	0.00	0.00	76.47	0.00	0.00	22.53	99.00	13.04
7	0.00	0.00	63.39	0.00	0.00	35.61	99.00	12.35	0.00	0.00	74.61	0.00	0.00	24.39	99.00	12.94
8	0.00	0.00	67.51	0.00	0.00	31.49	99.00	12.44	0.00	0.00	74.05	0.00	0.00	24.95	99.00	12.96
9	0.00	0.00	68.37	0.00	0.00	30.63	99.00	12.48	0.00	0.00	73.59	0.00	0.00	25.41	99.00	12.92
10	0.00	0.00	67.29	0.00	0.00	31.71	99.00	12.51	0.00	0.00	73.28	0.00	0.00	25.72	99.00	12.97
								γ=6								
1	0.00	0.00	24.74	0.00	0.00	43.64	68.38	8.38	0.00	0.00	35.62	0.00	0.00	39.79	75.41	9.34
2	0.00	0.00	26.02	0.00	0.00	43.30	69.31	8.41	0.00	0.00	34.49	0.00	0.00	40.41	74.89	9.23
3	0.00	0.00	26.57	0.00	0.00	42.13	68.69	8.46	0.00	0.00	35.65	0.00	0.00	37.97	73.62	9.16
4	0.00	0.00	27.13	0.00	0.00	42.10	69.23	8.55	0.00	0.00	34.83	0.00	0.00	41.46	76.29	9.27
5	0.00	0.00	29.49	0.00	0.00	39.98	69.47	8.62	0.00	0.00	34.52	0.00	0.00	40.10	74.62	9.16
6	0.00	0.00	28.85	0.00	0.00	43.89	72.75	8.77	0.00	0.00	35.63	0.00	0.00	40.00	75.63	9.21
7	0.00	0.00	28.65	0.00	0.00	41.74	70.38	8.71	0.00	0.00	33.85	0.00	0.00	40.19	74.04	9.12
8	0.00	0.00	31.12	0.00	0.00	39.73	70.84	8.77	0.00	0.00	34.10	0.00	0.00	39.99	74.09	9.13
9	0.00	0.00	31.51	0.00	0.00	41.36	72.87	8.82	0.00	0.00	32.99	0.00	0.00	39.64	72.63	9.07
10	0.00	0.00	30.67	0.00	0.00	40.85	71.52	8.81	0.00	0.00	32.84	0.00	0.00	40.56	73.40	9.11
								γ=9								
1	0.00	0.00	16.38	0.00	0.00	29.12	45.51	7.12	0.00	0.00	23.63	0.00	0.00	26.62	50.24	7.75
2	0.00	0.00	17.14	0.00	0.00	28.89	46.03	7.13	0.00	0.00	22.78	0.00	0.00	27.09	49.87	7.67
3	0.00	0.00	17.48	0.00	0.00	28.19	45.66	7.16	0.00	0.00	23.38	0.00	0.00	25.64	49.03	7.62
4	0.00	0.00	17.74	0.00	0.00	28.17	45.91	7.22	0.00	0.00	22.78	0.00	0.00	27.78	50.56	7.68
5	0.00	0.00	19.27	0.00	0.00	26.61	45.88	7.26	0.00	0.00	22.55	0.00	0.00	26.84	49.39	7.61
6	0.00	0.00	18.71	0.00	0.00	29.26	47.97	7.35	0.00	0.00	23.16	0.00	0.00	26.69	49.84	7.63
7	0.00	0.00	18.43	0.00	0.00	27.89	46.32	7.31	0.00	0.00	22.02	0.00	0.00	26.94	48.96	7.57
8	0.00	0.00	20.07	0.00	0.00	26.57	46.64	7.34	0.00	0.00	22.02	0.00	0.00	26.80	48.82	7.57
9	0.00	0.00	20.23	0.00	0.00	27.54	47.76	7.36	0.00	0.00	21.35	0.00	0.00	26.64	47.99	7.53
10	0.00	0.00	19.65	0.00	0.00	27.43	47.08	7.36	0.00	0.00	21.11	0.00	0.00	27.29	48.39	7.55

Table 9 - Continued

Panel B: The Case of Quarterly Observations

Horizon	SL	SM	SH	BL	BM	BH	Total	CE	SL	SM	SH	BL	BM	BH	Total	CE
	z_T=End-of-Sample Realizations								z_T=Sample Means							
								γ=3								
1	0.00	0.00	16.84	0.00	0.00	82.16	99.00	9.86	0.00	0.00	44.48	0.00	0.00	54.52	99.00	12.02
2	0.00	0.00	20.51	0.00	0.00	78.49	99.00	10.09	0.00	0.00	46.97	0.00	0.00	52.03	99.00	12.07
3	0.00	0.00	25.98	0.00	0.00	73.02	99.00	10.29	0.00	0.00	44.91	0.00	0.00	54.09	99.00	12.08
4	0.00	0.00	28.23	0.00	0.00	70.77	99.00	10.45	0.00	0.00	45.65	0.00	0.00	53.35	99.00	12.06
5	0.00	0.00	29.27	0.00	0.00	69.73	99.00	10.50	0.00	0.00	45.09	0.00	0.00	53.91	99.00	11.99
6	0.00	0.00	29.12	0.00	0.00	69.88	99.00	10.64	0.00	0.00	44.25	0.00	0.00	54.75	99.00	11.91
7	0.00	0.00	32.84	0.00	0.00	66.16	99.00	10.82	0.00	0.00	41.91	0.00	0.00	57.09	99.00	11.80
8	0.00	0.00	33.07	0.00	0.00	65.93	99.00	10.85	0.00	0.00	43.00	0.00	0.00	56.00	99.00	11.82
9	0.00	0.00	33.44	0.00	0.00	65.56	99.00	10.95	0.00	0.00	42.99	0.00	0.00	56.01	99.00	11.78
10	0.00	0.00	33.79	0.00	0.00	65.21	99.00	10.93	0.00	0.00	42.15	0.00	0.00	56.85	99.00	11.78
								γ=6								
1	0.00	0.00	7.07	0.00	0.00	45.16	52.23	7.22	0.00	0.00	16.60	0.00	0.00	46.13	62.73	8.41
2	0.00	0.00	8.34	0.00	0.00	44.72	53.06	7.33	0.00	0.00	17.97	0.00	0.00	43.80	61.77	8.41
3	0.00	0.00	10.67	0.00	0.00	42.67	53.35	7.42	0.00	0.00	16.55	0.00	0.00	45.45	62.00	8.40
4	0.00	0.00	11.66	0.00	0.00	41.89	53.55	7.49	0.00	0.00	16.92	0.00	0.00	45.39	62.31	8.40
5	0.00	0.00	11.66	0.00	0.00	42.29	53.95	7.51	0.00	0.00	16.50	0.00	0.00	45.39	61.89	8.34
6	0.00	0.00	11.32	0.00	0.00	42.92	54.24	7.57	0.00	0.00	16.08	0.00	0.00	44.96	61.04	8.28
7	0.00	0.00	12.61	0.00	0.00	42.82	55.44	7.66	0.00	0.00	15.11	0.00	0.00	44.78	59.89	8.20
8	0.00	0.00	12.50	0.00	0.00	42.86	55.36	7.66	0.00	0.00	15.53	0.00	0.00	44.47	60.00	8.20
9	0.00	0.00	12.48	0.00	0.00	42.99	55.47	7.71	0.00	0.00	15.68	0.00	0.00	43.69	59.36	8.17
10	0.00	0.00	12.94	0.00	0.00	41.44	54.38	7.69	0.00	0.00	15.13	0.00	0.00	44.91	60.04	8.17
								γ=9								
1	0.00	0.00	4.65	0.00	0.00	30.07	34.72	6.36	0.00	0.00	10.92	0.00	0.00	30.81	41.73	7.14
2	0.00	0.00	5.41	0.00	0.00	29.78	35.19	6.42	0.00	0.00	11.73	0.00	0.00	29.28	41.01	7.13
3	0.00	0.00	6.87	0.00	0.00	28.42	35.29	6.48	0.00	0.00	10.72	0.00	0.00	30.34	41.06	7.12
4	0.00	0.00	7.49	0.00	0.00	27.88	35.37	6.52	0.00	0.00	10.90	0.00	0.00	30.33	41.23	7.11
5	0.00	0.00	7.42	0.00	0.00	28.13	35.55	6.53	0.00	0.00	10.51	0.00	0.00	30.33	40.84	7.07
6	0.00	0.00	7.18	0.00	0.00	28.51	35.69	6.57	0.00	0.00	10.18	0.00	0.00	30.03	40.21	7.02
7	0.00	0.00	7.96	0.00	0.00	28.43	36.39	6.62	0.00	0.00	9.54	0.00	0.00	29.83	39.37	6.97
8	0.00	0.00	7.86	0.00	0.00	28.40	36.26	6.62	0.00	0.00	9.78	0.00	0.00	29.62	39.40	6.96
9	0.00	0.00	7.79	0.00	0.00	28.50	36.28	6.64	0.00	0.00	9.84	0.00	0.00	29.13	38.97	6.94
10	0.00	0.00	8.11	0.00	0.00	27.49	35.61	6.63	0.00	0.00	9.45	0.00	0.00	29.85	39.30	6.94