Consistent Return Estimates - The Black-Litterman Approach.

Presentation for

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BLACK-LITTERMAN.
THE APPROACH.

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Consistent asset return estimates - saving classical mean/variance...

In asset management, the forecast of asset returns is essential within the investment process. In this context, the Black-Litterman approach (1992) yields consistent asset return forecasts as a weighted combination of (strategic) market equilibrium returns and (tactical) subjective forecasts (”views”). The Black-Litterman formalism allows to implement both absolute views (return levels) and relative views (outperforming vs. underperforming assets) for selected assets investigated under „core competence“. For any particular view, individual confidence levels for the return estimates have to be specified. The formalism spreads these informations consistently across all assets in the portfolio. The BL-revised returns then serve as a consistent input for mean-variance portfolio optimization procedures, thus allowing for the implementation of additional constraints. BL-optimized portfolios overcome some well-known Markowitz/MV insufficiencies as unrealistic sensitivity to input factors, extreme portfolio weights and excessive turnovers. The BL process will be introduced both from its theoretical background and its implementation in practice.

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OPINIONS AND PORTFOLIO CONTEXT.

RESEARCH

- The c.p. world of core competences -

Opinion
Opinion
Opinion Opinion
Opinion
Opinion
Opinion
Opinion

?

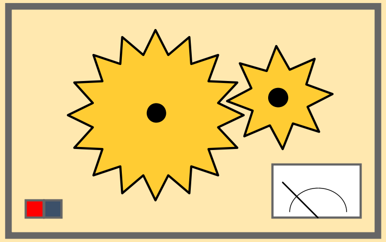
PORTFOLIO CONSTRUCTION

- The portfolio context: Thinking in terms of correlations -

Committee

or

Quant tool



The nice-to-haves

„Consistent" (non-c.p.) input for portfolio construction

Intuitive results

Tactical deviations from „some" strategic allocation

Transparent

Managable

Reliable output

Overcome some of the problems of plain MV (Markowitz)

Weighting estimates according to confidence

Black-Litterman

AGENDA.
ASPECTS TO BE DISCUSSED.

- > Classical Markowitz
- > MV-optimized portfolios - the straight way
- > BL - part within the investment process
- > BL - implementation
- > BL - example

1952 Markowitz
(Portfolio Selection)

1964 Sharpe
(CAPM)

1976 Ross
(APT)

1990 Black
(Universal Hedging)

1991 Black-Litterman

MARKOWITZ APPROACH.
REVISITED.

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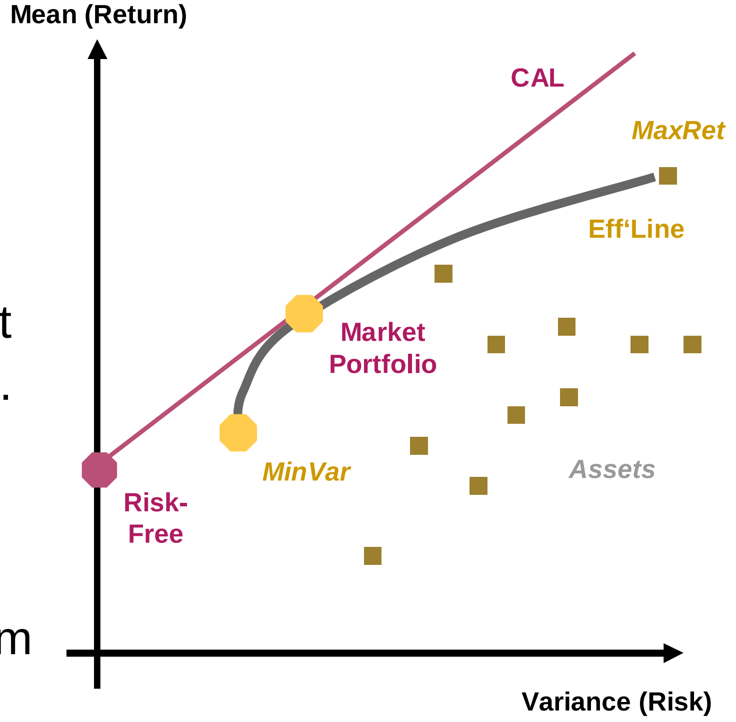
Efficient portfolios in the mean/variance framework

> Starting point in a world of normally distributed returns: The assets are described by the first two moments of return - mean and variance.

> In an efficient portfolio the assets are weighted such that for any given level of risk a maximum return is achieved. (equivalently: for a given return the risk is minimized). Diversification reduces risk.

> All efficient portfolios form the efficiency line. It starts in the minimum variance portfolio and ends in the maximum return portfolio (which is the asset of maximum return).



> If a risk-free asset exists, all efficient portfolios are located on the Capital Allocation Line (CAL), starting at the risk-free asset and tangentially touching the efficiency line at the market portfolio. Efficient portfolios are then a combination of the risky market portfolio and the risk-free asset (with risk-free *long* or *short*), a.k.a. *Tobin Separation (1958)*.

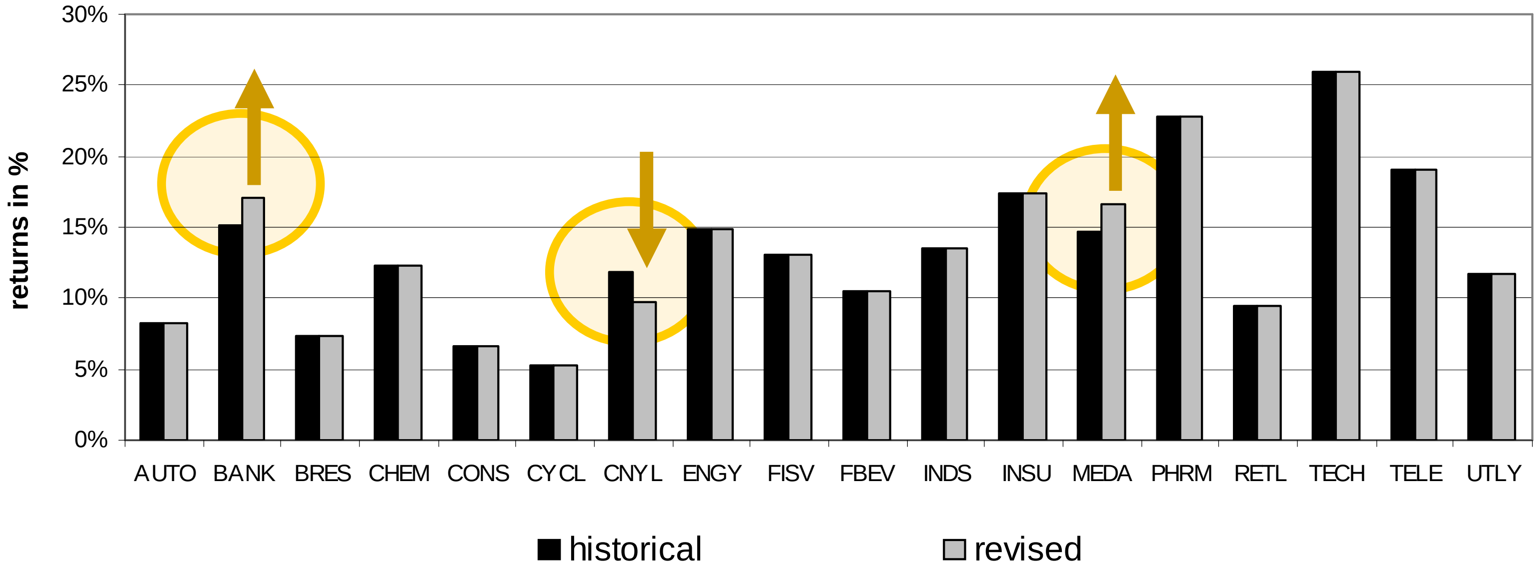
MARKOWITZ APPROACH.
DEFICITS AND WORKAROUNDS.

Consistent asset return estimates - saving classical mean/variance...

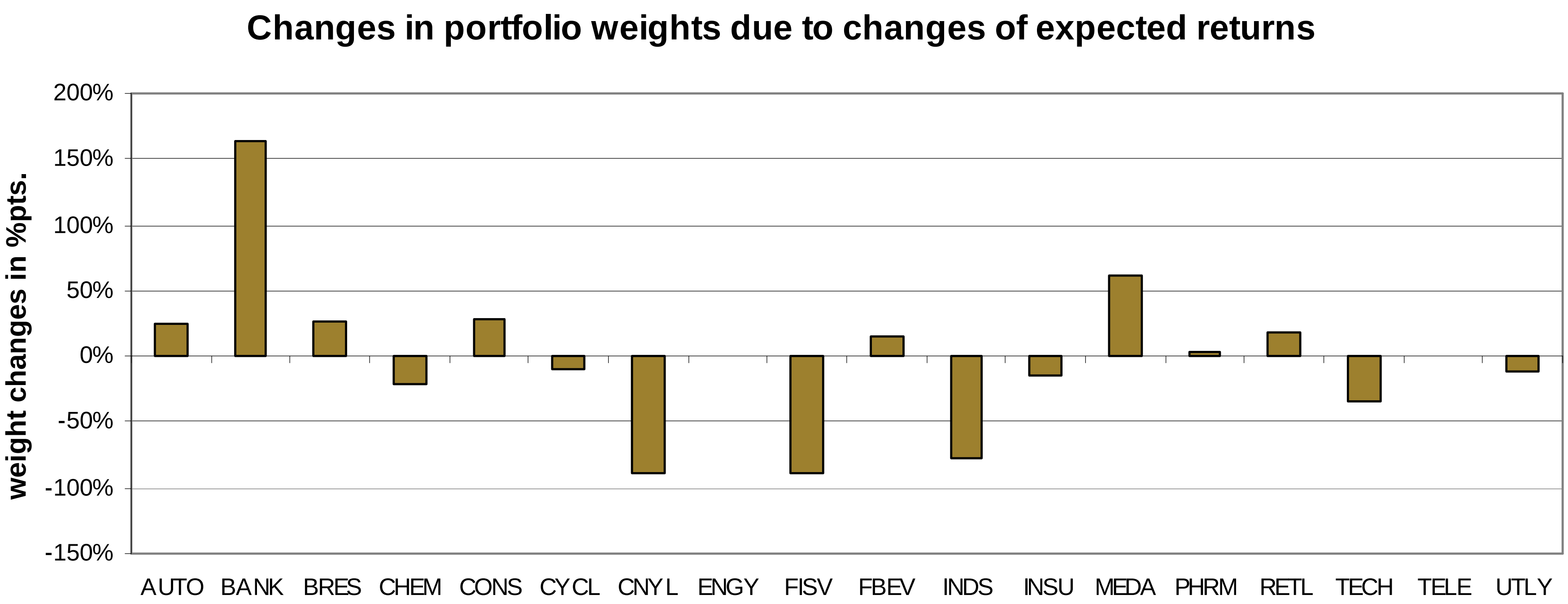
Deficit	Improvement
> <u>High sensitivity on inputs</u> (return estimates!) leads to large weight fluctuations in the optimal portfolio.	*Black-Litterman*
> „Corner solutions“: <u>Extreme portfolio weights</u> (also in the case of optimization algorithms using constraints)	*Black-Litterman*
> <u>Aggregation:</u> Consistent aggregation of huge number of estimated returns overburdens the investment process	*Black-Litterman*
No quantification of <u>confidence</u> in estimated returns	*Black-Litterman*
> One-periodical approach	Multi-period appr., ...
> „Variance“= restricting risk to symmetric return volatility	VaR, ...
> Requires ex-ante-estimates of covariance matrix	Vola-modeling, ...
> ...	

- Let the investment universe be the 18 DJ STOXX sectors.
- **Today**: Returns : Historical returns
  Weights : To be determined via MV (mean/variance optimization)
- **Forecast**: Returns : Revised: +2%pts for **BANK** and **MEDA**, -2%pts for **CNYL**
  Weights : To be determined via MV

Historical returns and revised expected returns



> Observation: Even small and selected changes in expected returns lead to huge unrealistic shifts in **asset weights!** **(γ=3, historical covariances, no constraints)**



> Problems: Communication of results, (re-)allocation in real portfolios, acceptance of method.

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The formal MV-optimization approach, basic outline.

> Markowitz theory relates risk & return

> $\Rightarrow$ MV optimization problem:

$$w^{\mathrm{T}} R - \frac{\gamma}{2} \cdot w^{\mathrm{T}} \Omega\, w \rightarrow \max_{w}$$

R = *vector of returns*
Ω = *covariance matrix*
γ = *risk aversion parameter*
w = *vector of weights*

> $\Rightarrow$ Solution for the optimal portfolio weights w^* (no constraints):

$$w^* = (\gamma\, \Omega)^{-1} R$$

> Given $\gamma\, \Omega$ and *R* this is an *exact* formalism to achieve optimal (efficient) portfolios.

What about „garbage in - garbage out"?

MARKOWITZ APPROACH - EXTENDED.
EQUILIBRIUM RETURNS.

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Supply & demand

- Traditional approach of maximum return & minimum risk is demand-side perspective only.
- Need to balance with supply-side...

Concept of equilibrium returns - reverse optimization:

- The market portfolio reflects market equilibrium, i.e. supply & demand are balanced.
- Therefore, equilibrium returns reflect neutral „fair“ reference returns Π :
- Formally, reverse optimization yields equilibrium returns:

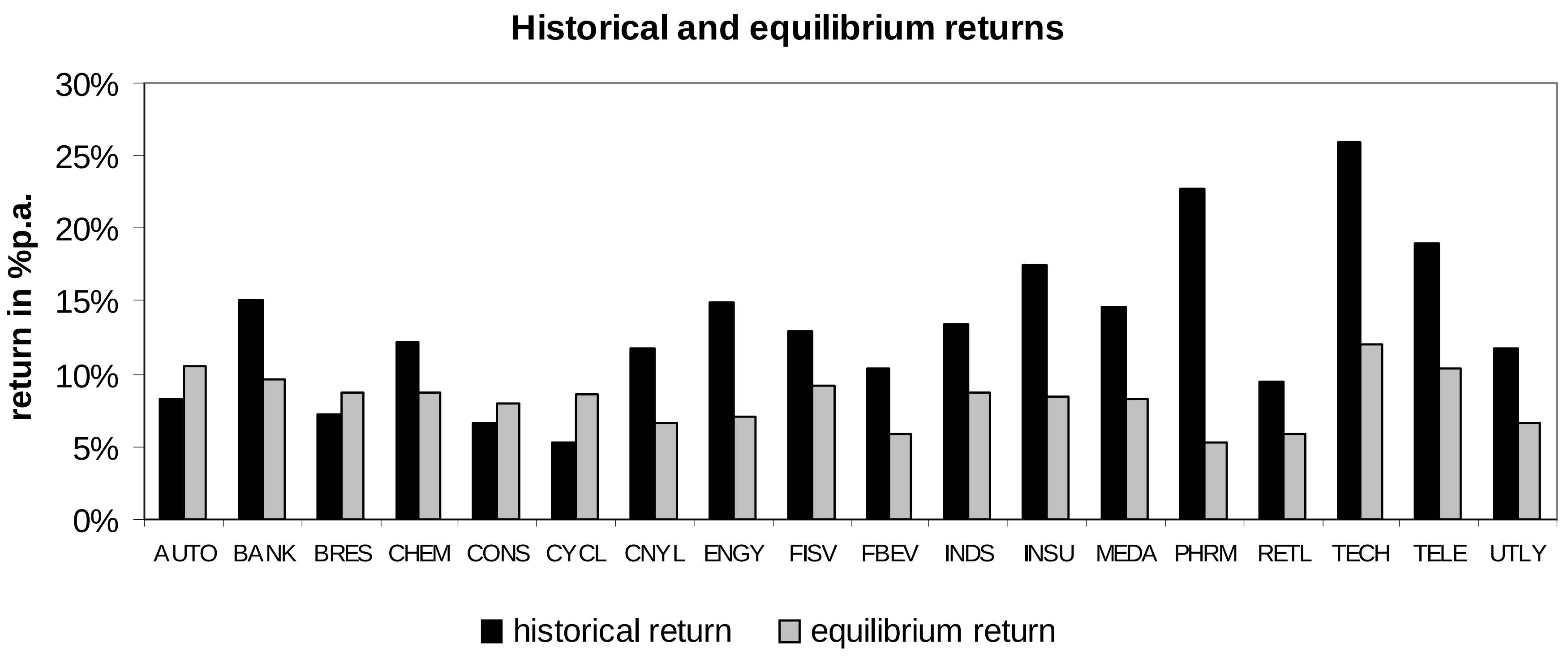
$$\Pi = (\gamma \, \Omega) \, w_{\text{MCap}}$$

w_{MCap} = *market capitalization*

Conclusion

- Use of equilibrium returns as market neutral reference.
- Market portfolio can be approximated by the investment universe, e.g. the Benchmark

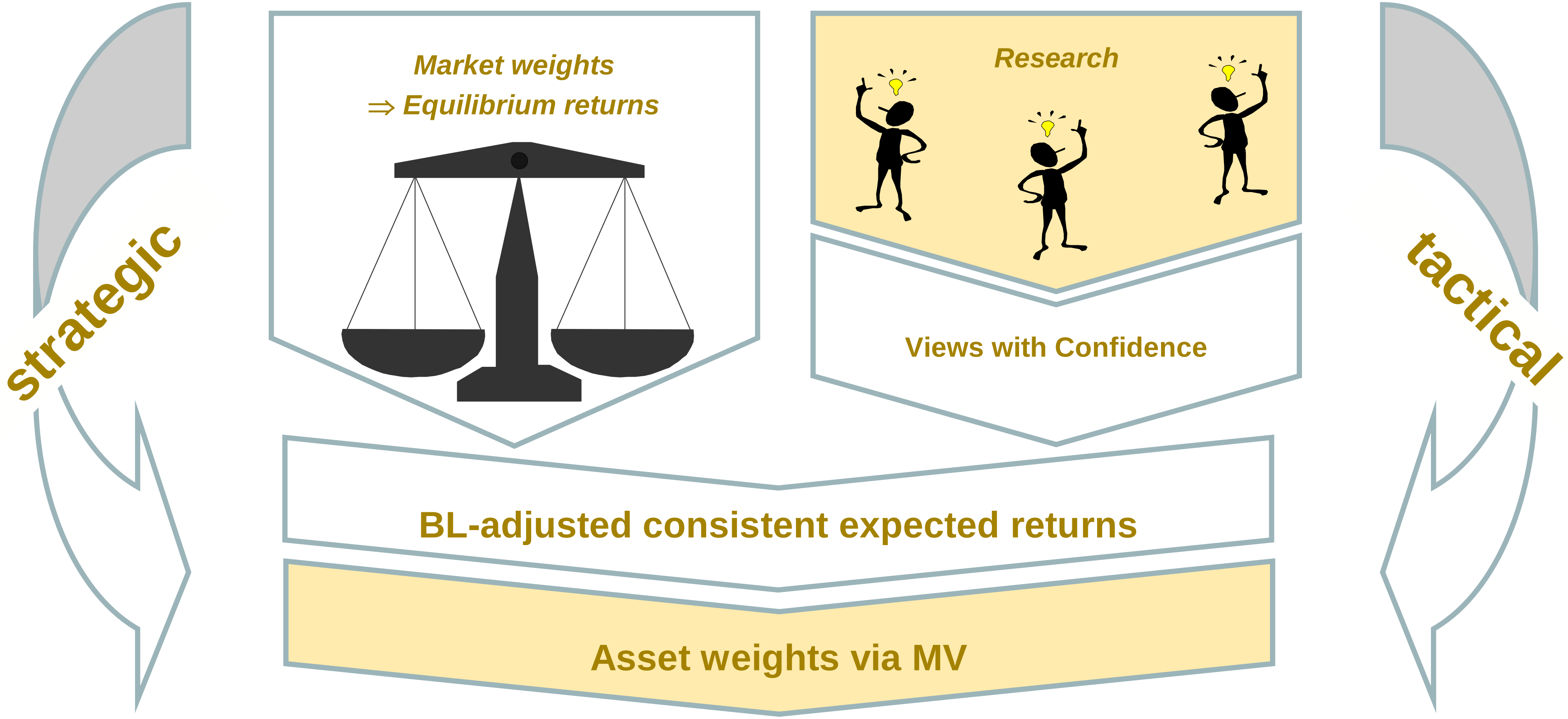
MARKOWITZ APPROACH - EXTENDED.
EQUILIBRIUM RETURNS FOR THE STOXX SECTORS.



> These equilibrium returns serve as reference returns for all further investigations.
> Note that equilibrium returns are *calculated*; they do not require any estimate.

BLACK-LITTERMAN APPROACH.
THE FRAME.

BL is a conceptual approach to combine long term market equilibrium returns with subjective short term return estimates to get consistent returns for MV-optimization.



BLACK-LITTERMAN APPROACH.
OPTIMIZATION.

> Determine the optimal estimate E(R) which minimizes the variance of E(R) w.r.t. equilibrium returns Π (minimizing the Mahalanobis distance):

$$\left[E(R)-\Pi\right]^{\mathrm{T}} \cdot \left(\tau\,\Omega\right)^{-1} \cdot \left[E(R)-\Pi\right] \rightarrow \min_{E(R)}$$

where: $E(R) = \Pi + \nu \quad \text{with} \quad \nu \sim N(0, \tau\,\Omega).$

s.t.

$$P \cdot E(R) = \begin{cases} V & \text{certain Views} \\ V + e & \text{uncertain Views} \end{cases}$$

P = View portfolios

where: $P \cdot E(R) \sim \mathrm{N}(V, \Sigma)$, $(\Sigma)_{ij} = \delta_{ij}\, e_i$ (detailed discussion follows, see also app.)

THE MASTER FORMULA.

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$$\overline{E}(R) = \left[(\tau\Omega)^{-1} + P^T\Sigma^{-1}P\right]^{-1} \times \left[(\tau\Omega)^{-1}\cdot\Pi + P^T\Sigma^{-1}\underbrace{P\cdot P^{-1}}_{=\mathbb{1}}V\right]$$

> Complex interaction between equilibrium returns and subjective return estimates.

> First factor („Denominator“): Normalisation.

> Second factor („Numerator“): Balance between Π (=equilibrium returns) and $\mathbf{P^{-1}V}$ (=Views).
Inverse of covariance $(\tau\,\Omega)^{-1}$ and the confidence $P^T\Sigma^{-1}P$ serve as weighting factors.

> Constituents: Matrix $\tau\,\Omega$: covariance of historical returns, $\tau =$ overall weight parameter
Matrix *P* : formal aggregation of View portfolios
Matrix Σ^{-1} : confidence in Views (Σ = „covariance of estimated Views“)
Σ assumed to be diagonal, i.e. no cross-informations on Views.

BLACK-LITTERMAN APPROACH.
STILL UNDER INVESTIGATION.

- Calibration of parameter τ ("0.3 is plausible", "adjusted to IR=1", ...)
- Calibration of parameter γ ("world wide risk aversion", ...)
- Calibration of degree of confidence ("1..3", "0..100%")

- Only few publications available about implementation and/or experience

Views

> **Tactical return estimates differing from the (strategic) equilibrium returns are the essential input to the BL estimation process.**

Specification of Views

> ... as absolute return expectations for individual assets
> *and / or*
> ... as relative return expectations relating assets or aggregates of assets.
> Formal constraint: #Views ≤ #Assets.

Confidence in Views

> Each View has to be assigned the level of confidence (for an interval of uncertainty).

Focus of selected Views

> Views can be restricted to selected assets for which in-depth analysis is available.

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Formal aggregation of different Views...

Relative and absolute Views are formally aggregated to a system of linear equations:

$$P \cdot E(R) = V + e$$

where (k = #Views and n = #Assets, with k ≤ n):

- E(R) = n×1 vector of expected asset returns, unknown
- P = k×n matrix, weighting the asses
- V = k×1 vector, absolute or relative return expectations (i.e., levels or over-/underperforming figures)
- e = k×1 vector of squared StDev‘s (note that Σ^{-1} is a k×k diagonal matrix expressing confidence, assuming independent estimation errors, with $\Sigma_{ii} = e_i$)

This relation is formally integrated in the BL master equation.

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> A relative View can be stated as follows: „The sectors Pharmacy and Industry will outperform Telecom and Technology by 3% ± 1% with a confidence of 90%“:

$$\left[w_{PHRM}\cdot E(R_{PHRM})+w_{INDU}\cdot E(R_{INDU})\right]-\left[w_{TELE}\cdot E(R_{TELE})+w_{TECH}\cdot E(R_{TECH})\right]=3\%+(0.61\%)^2$$

> View portfolios: A *long*-portfolio with outperformers, a *short*-portfolio with underperformers.

> View portfolio weights: $$\left[w_{PHRM}+w_{INDU}\right]=100\%=\left[w_{TELE}+w_{TECH}\right]$$

> An absolute View can be stated as follows: „The sector of Non-Cyclical Goods (CNYL) will perform better than the equilibrium return of 6.66%. Our new target return is 7.5% with 90% of confidence within a range of ±1.5%“:

$$w_{CNYL}\cdot E(R_{CNYL})=7.5\%+(0.91\%)^2$$

> View portfolio weight: $$w_{CNYL}=100\%$$

BLACK-LITTERMAN APPROACH.
COMBINED VIEWS - EXPLICIT.

> Combining the aforementioned Views using $P \cdot E = V + e$, we get:

$$P \cdot \begin{pmatrix} E(R_{AUTO}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = \begin{pmatrix} 3\% \\ 7.5\% \end{pmatrix} + \begin{pmatrix} (0.61\%)^2 \\ (0.91\%)^2 \end{pmatrix}$$

> The View-related portfolios (weights) are given by:

long positions (0.34, 0.66) — *short* positions (-0.51, -0.49)

$$P = \begin{pmatrix} View\,1, rel. \\ View\,2, abs. \end{pmatrix} = \begin{pmatrix} 0 & \cdots 0 & \cdots & 0 & 0.34 & 0 & 0 & 0.66 & 0 & -0.51 & -0.49 & 0 \\ 0 & \cdots & 1 & 0 & & \cdots & & \cdots & & \cdots & & 0 \end{pmatrix}$$

AUTO ⋮ CNYL ENGY FISV FBEV INDS INSU MEDA PHRM RETL TECH TELE UTLY

long position (CNYL)

Sector Allocation, Dow Jones STOXX

- Example follows the lines of
  „Einsatz des Black-Litterman-Verfahrens in der Asset Allocation“,
  H.Zimmermann et al. publ. in „Handbuch Asset Allocation“
  (Editors: Dichtl, Schlenger u. Kleeberg, publ. by Uhlenbruch-Verlag, 2002).
- Notation, scenarios and data therein have been used, some data were missing.
- Missing data - volatilities and covariances - had to be calculated, thus causing some deviations in the numerical results between this presentation and cited literature.

  Nevertheless, all relevant results are reproduced.
- All calculations can be (have been) implemented and performed in Excel (TM).

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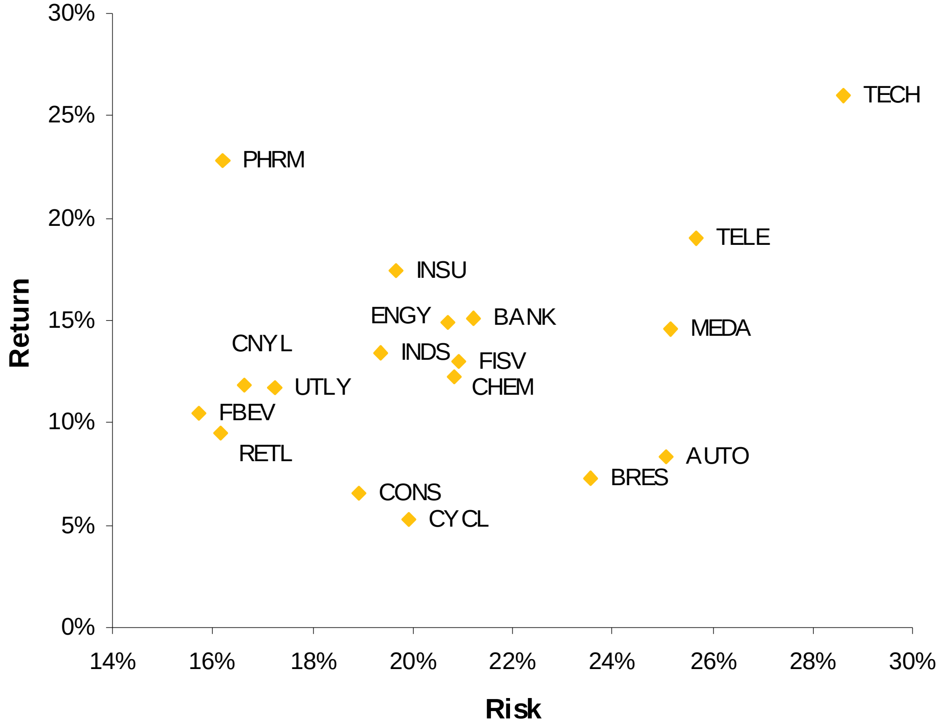
18 Sectors in the Dow Jones STOXX index

> Monthly returns in Sfr (Swiss francs), period: 06/1993 - 11/2000, annualized data.

Sector	hist.Return	hist.Volatility	MarketCap
total:	**average:**		**total:**
18	**16,22%**		**100,01%**
AUTO	8,32%	25,09%	1,65%
BANK	15,14%	21,21%	15,04%
BRES	7,31%	23,56%	1,22%
CHEM	12,25%	20,81%	1,80%
CONS	6,56%	18,92%	1,26%
CYCL	5,24%	19,94%	2,85%
CNYL	11,80%	16,66%	2,90%
ENGY	14,92%	20,72%	10,30%
FISV	13,01%	20,91%	4,12%
FBEV	10,47%	15,72%	4,59%
INDS	13,45%	19,35%	5,19%
INSU	17,43%	19,68%	6,89%
MEDA	14,63%	25,17%	3,27%
PHRM	22,83%	16,20%	10,24%
RETL	9,49%	16,16%	2,27%
TECH	25,95%	28,60%	11,03%
TELE	18,99%	25,69%	10,56%
UTLY	11,77%	17,25%	4,83%

Historical Data - Risk/Return Characteristics



Calculations based on monthly returns

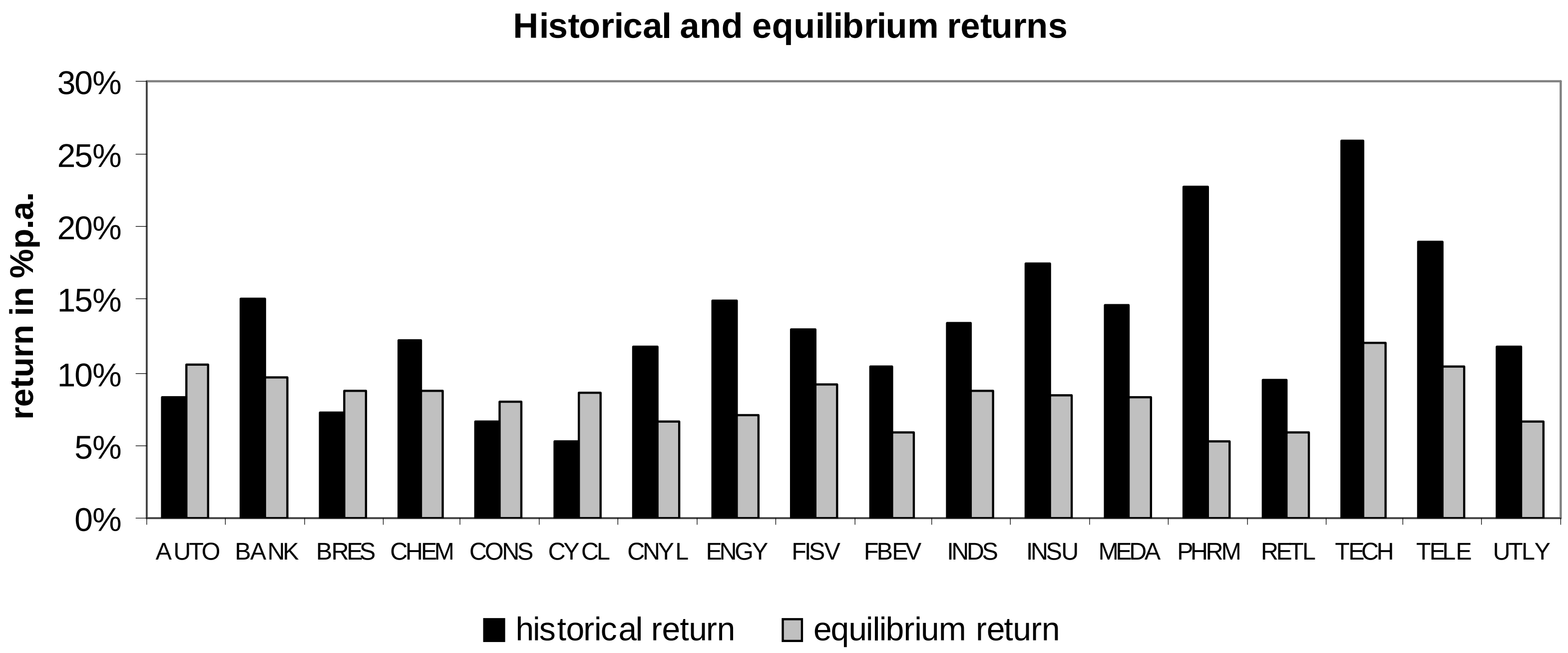
> Correlation of Dow Jones STOXX sectors

	AUTO	BANK	BRES	CHEM	CONS	CYCL	CNYL	ENGY	FISV	FBEV	INDS	INSU	MEDA	PHRM	RETL	TECH	TELE	UTLY
AUTO	**100%**	**74%**	**73%**	**83%**	**78%**	**75%**	**73%**	55%	**73%**	**71%**	**79%**	**72%**	46%	43%	68%	69%	65%	64%
BANK	**74%**	**100%**	63%	**73%**	**74%**	**75%**	**71%**	59%	**92%**	**75%**	**75%**	**87%**	39%	63%	62%	67%	59%	64%
BRES	**73%**	63%	**100%**	**83%**	**81%**	**78%**	60%	66%	69%	56%	**78%**	56%	44%	31%	59%	62%	52%	41%
CHEM	**83%**	**73%**	**83%**	**100%**	**85%**	**82%**	**72%**	67%	**72%**	**74%**	**82%**	70%	51%	45%	69%	65%	57%	57%
CONS	**78%**	**74%**	**81%**	**85%**	**100%**	**90%**	**72%**	66%	**75%**	**76%**	**89%**	64%	54%	39%	67%	66%	63%	64%
CYCL	**75%**	**75%**	**78%**	**82%**	**90%**	**100%**	67%	64%	**79%**	70%	**87%**	63%	58%	43%	67%	**70%**	63%	56%
CNYL	**73%**	**71%**	60%	**72%**	**72%**	67%	**100%**	55%	69%	**75%**	69%	**74%**	41%	58%	**73%**	53%	59%	**71%**
ENGY	55%	59%	66%	67%	66%	64%	55%	**100%**	59%	59%	59%	54%	28%	43%	55%	43%	32%	46%
FISV	**73%**	**92%**	69%	**72%**	**75%**	**79%**	69%	59%	**100%**	**75%**	**73%**	**85%**	39%	61%	58%	64%	57%	58%
FBEV	**71%**	**75%**	56%	**74%**	**76%**	70%	**75%**	59%	**75%**	**100%**	62%	**74%**	27%	63%	61%	40%	41%	66%
INDS	**79%**	**75%**	**78%**	**82%**	**89%**	**87%**	69%	59%	**73%**	62%	**100%**	65%	**72%**	38%	68%	**82%**	**77%**	67%
INSU	**72%**	**87%**	56%	70%	64%	63%	**74%**	54%	**85%**	**74%**	65%	**100%**	36%	67%	61%	60%	56%	68%
MEDA	46%	39%	44%	51%	54%	58%	41%	28%	39%	27%	**72%**	36%	**100%**	21%	42%	**75%**	**77%**	57%
PHRM	43%	63%	31%	45%	39%	43%	58%	43%	61%	63%	38%	67%	21%	**100%**	43%	35%	37%	58%
RETL	68%	62%	59%	69%	67%	67%	**73%**	55%	58%	61%	68%	61%	42%	43%	**100%**	52%	53%	57%
TECH	69%	67%	62%	65%	66%	**70%**	53%	43%	64%	40%	**82%**	60%	**75%**	35%	52%	**100%**	**81%**	55%
TELE	65%	59%	52%	57%	63%	63%	59%	32%	57%	41%	**77%**	56%	**77%**	37%	53%	**81%**	**100%**	70%
UTLY	64%	64%	41%	57%	64%	56%	**71%**	46%	58%	66%	67%	68%	57%	58%	57%	55%	70%	**100%**

> Covariance matrix via $\Omega_{ij} = \sigma_i \sigma_j \rho_{ij}$.

BL-starting point: Equilibrium (or implicit) returns of market portfolio

> reverse optimization yields equilibrium returns: $\Pi = (\gamma\,\Omega) \cdot w_{MCap}$



From equilibrium returns to Black-Litterman returns

> **Relative View**

Sectors Pharmacy and Industry will outperform Tech and Tele by 3%.

> **Absolute View**

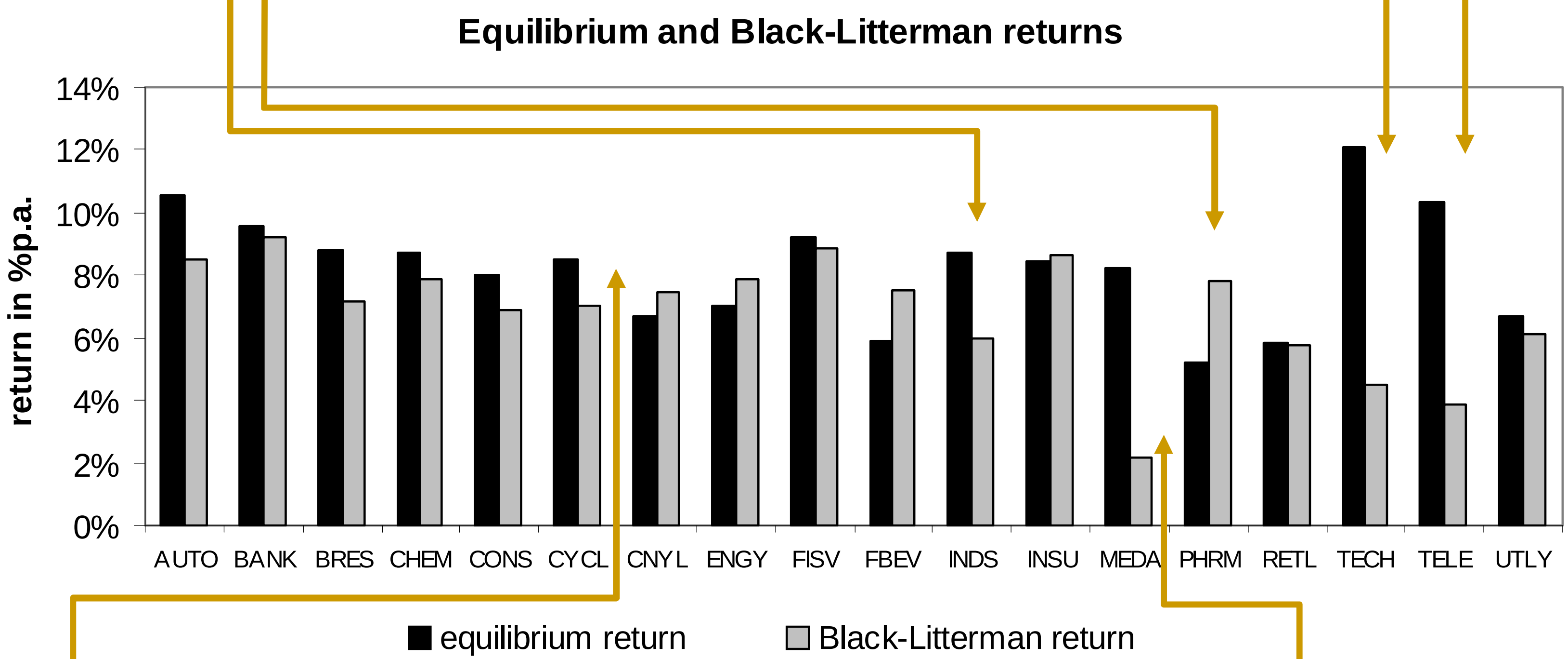
Non-Cyclical Goods will perform better at 7.5% (instead of 6.66%).

> **Note:** Only 5 out of 18 sectors with explicit Views, rest unchanged.

BLACK-LITTERMAN APPROACH.
APPLYING THE MASTER FORMULA → BL-RETURNS.

BL return expectation higher in PHRM but lower in INDS (note that„... better than TECH and TELE“ remains intact!)

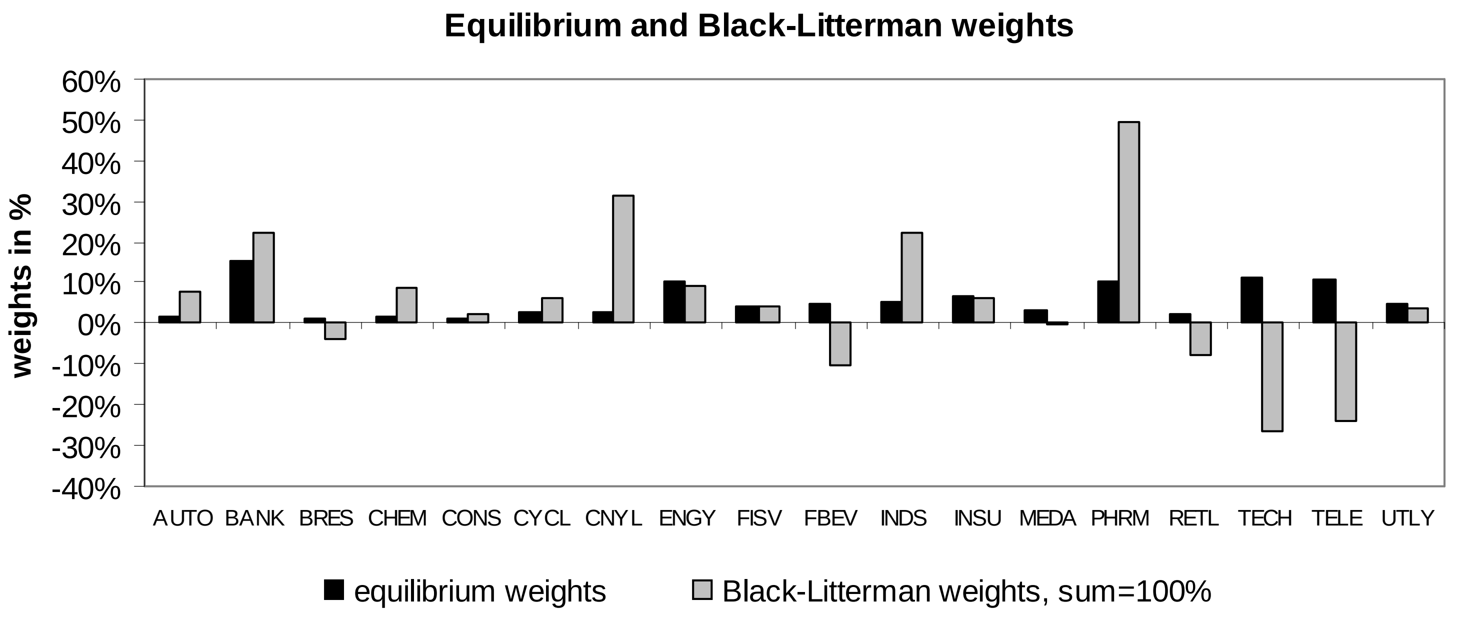
BL return expectations significantly lower for TECH and TELE.



For CNYL, the BL-expected return increases from 6.66% to 7.48% (90% confidence in target View of 7.5%).

Example: MEDA, no explicit View, correlation of 75% to TECH and 77% to TELE leads to significantly lower BL-return.

Comparing equilibrium weights (market cap.) and Black-Litterman weights



Using the BL returns (note: strong confidence in Views of 90%):

> Significant weight reduction for TECH and TELE (MEDA also reduced)

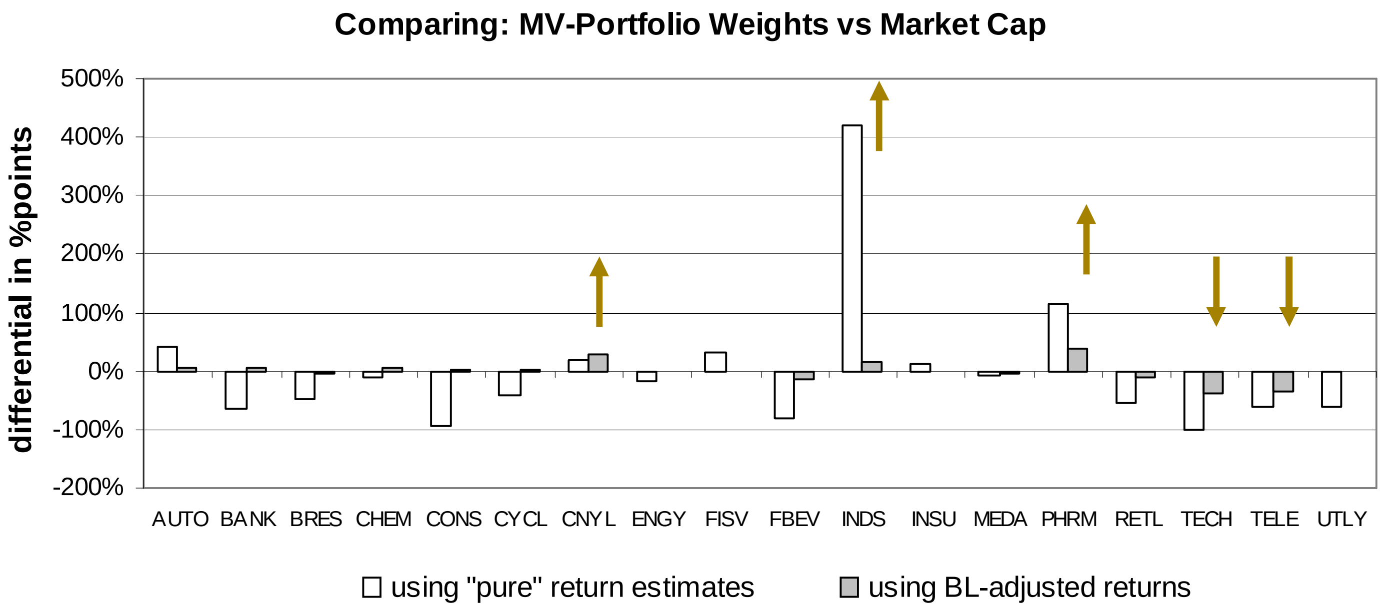
> Significant weight increase for INDS and PHRM

> Significantly increased weight for CNYL due to higher return expectation

COMPARING.

BLACK-LITTERMAN APPROACH vs. STRAIGHT MV.

Given the return scenario (Views up or down, ↑ ↓), the revised portfolio structure clearly benefits from the BL-enhanced optimization process.



> **Straight MV optimization** - which is a naiv approach in terms of „c.p.“ return estimates - yields extreme and unreliable changes in portfolio weights

> **The BL-adjusted return input for MV** stabilizes the weights, thus leading to a reliable and intuitively sound new portfolio structure.

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Remark on treating MV-weights w.r.t. absolute and relative Views

> **Purely absolute Views** are translated into independent *long* and *short* portfolios, thus causing MV portfolio weights to deviate from 100%. Therefore, normalization of weights in the MV optimization process is recommended.

> **Purely relative Views** are translated into weight-balanced *long* and *short* portfolios, so that portfolio weights still sum up to 100%.

> **The use of absolute** and **relative Views** again leads to MV portfolio weights deviating from 100%. Therefore, again, normalization of weights is recommended.

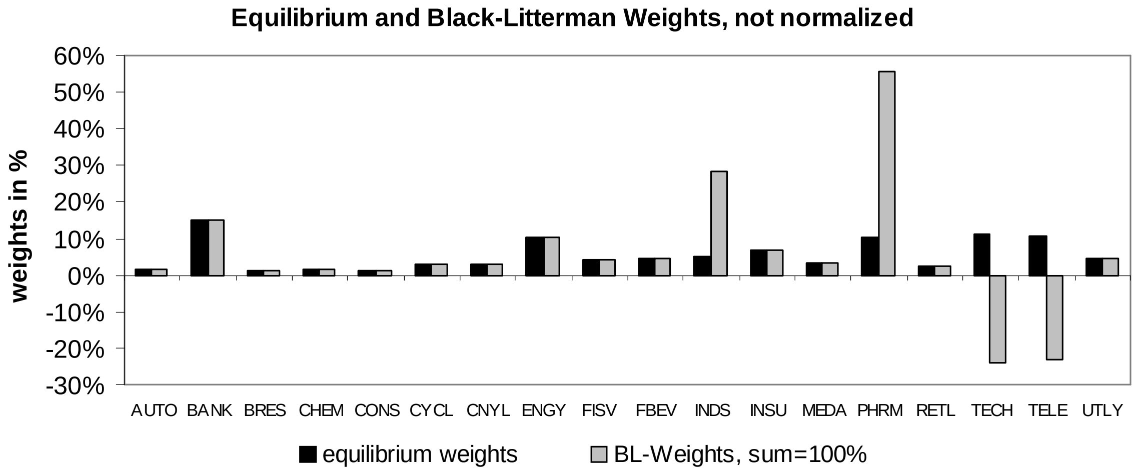
> **Budget constraint**: The sum of portfolio weights adds up to **100%** ($I = \mathbf{1}\text{-}Vector$) :

$$w = \frac{\Omega^{-1} I}{I^{\mathrm{T}} \Omega^{-1} I} + \frac{1}{\gamma} \Omega^{-1} \cdot \left(R - \frac{I^{\mathrm{T}} \Omega^{-1} R}{I^{\mathrm{T}} \Omega^{-1} I} I \right)$$

BLACK-LITTERMAN APPROACH.
WEIGHTS 1.

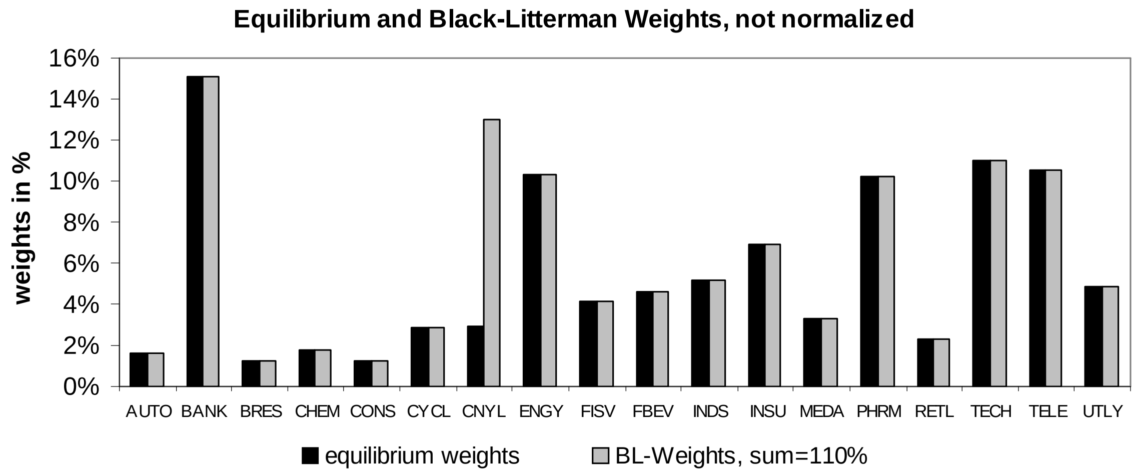
Only relative Views
sum of weights = 100%

> **no normalization required**



Only absolute View
sum of weights = 110%

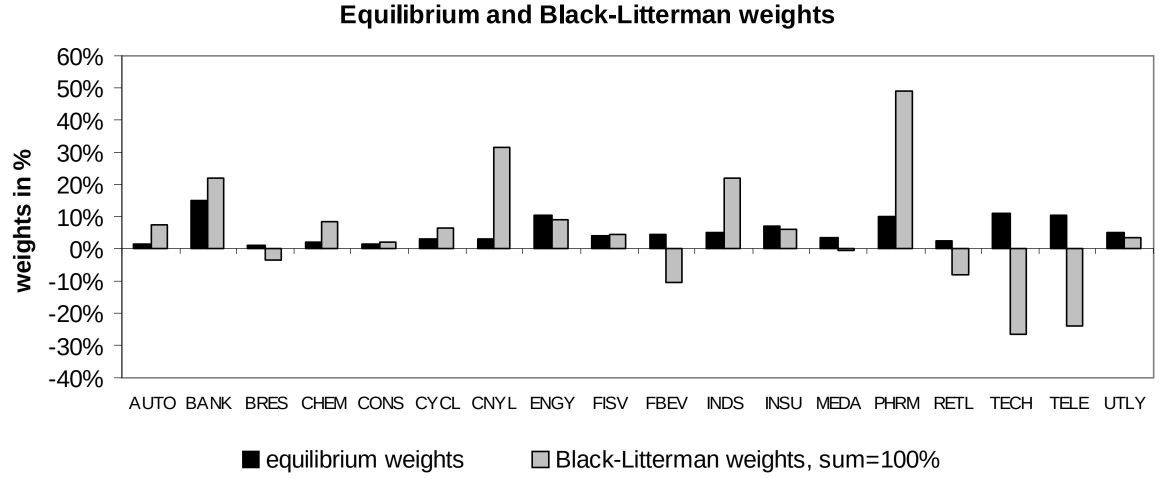
> **normalization recommended**



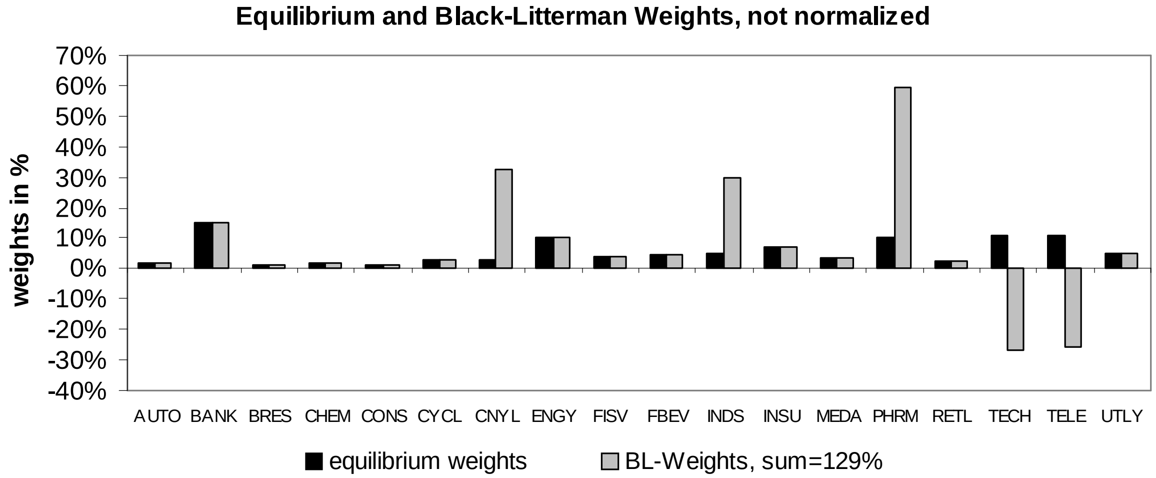
BLACK-LITTERMAN APPROACH.
WEIGHTS 2.

Relative and absolute Views

> **use normalized weights**
> **sum of weights = 100%**



> **weights not normalized**
> **sum of weights = 129%**



BLACK-LITTERMAN APPROACH.
WEIGHTS 3.

Final remark on non-normalized weights

- **Assets with particular View:** The weights differ from those of the equilibrium (market) portfolio, depending on the positive or negative Views.
- **Assets with no particular View:** The weights of these assets are **not** changed compared to the market portfolio - which is then regarded as the best positioning (the same holds traditionally for a long-only investor who replicates the benchmark if he has no specific idea on expected returns).
- The sum of weights exceeding 100% requires to borrow money for financing.

BLACK-LITTERMAN APPROACH.
CONFIDENCE.

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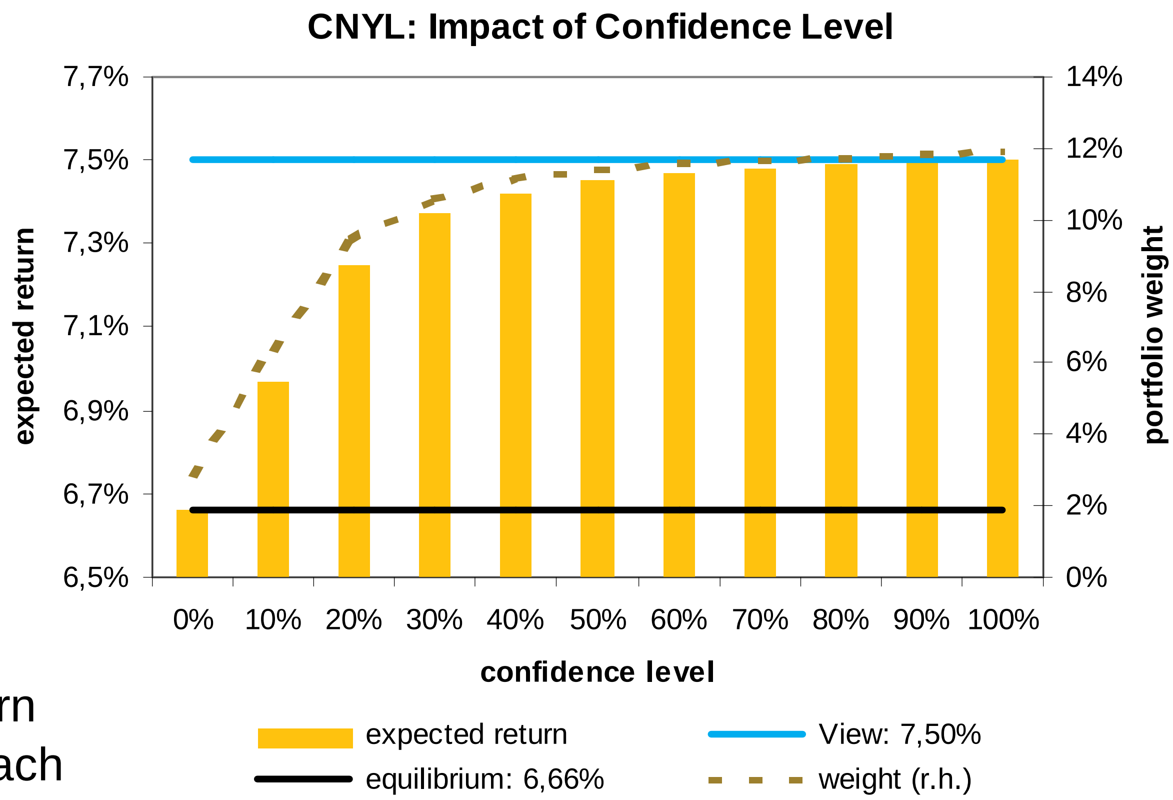
Focus on asset „CNYL“ only:

> Equilibrium return = 6.66%,
> Equilibrium weight = 2.90%
> View on return: 7.5% ± 1.5% (equivalent to a range of 6 - 9%)



Observations:

> Low confidence: → equilibrium return
> High confidence: Asymptotic approach to the View value of 7.5%.
> Limit: At a confidence level of 100%, BL fully accepts the strong View of 7.5%.
> Weights range from 2.9% (= market cap, due to confidence of 0% the *no View*-case) up to 12% (overweighting due to the *strong View* confidence of 100%).

BLACK-LITTERMAN APPROACH.
CONFIDENCE AND WEIGHTS.

Behavior of asset weights:

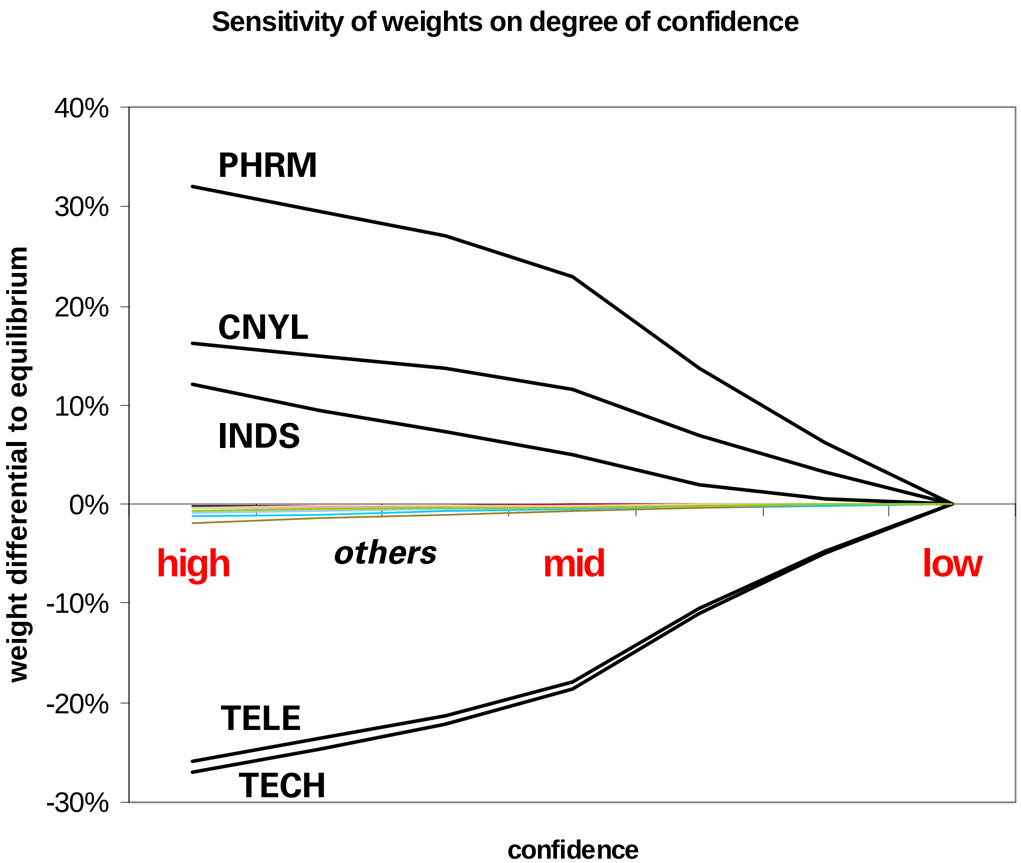
> Complete portfolio, 18 sectors

Observations:

> Low degrees of confidence: BL-weights are close to weights in equilibrium (=market cap's).

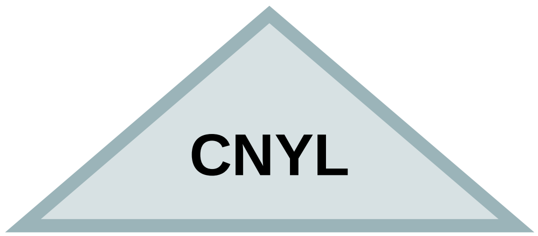
> Higher degree of confidence: Weights approach equilibrium values on either underweighting (*short*) or overweighting (*long*) path.

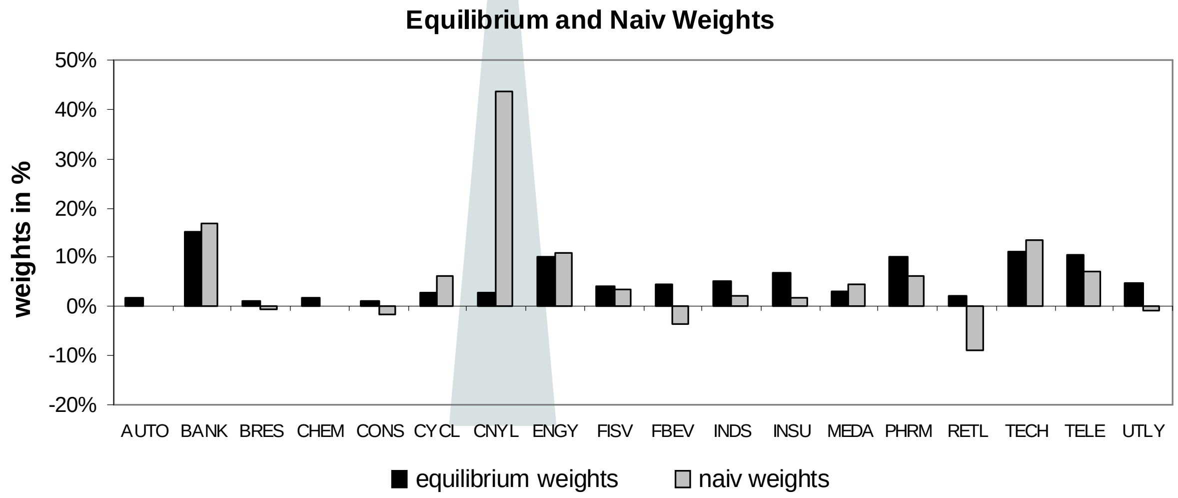
> **Significant weight changes only for the assets under View!**



SENSITIVITY OF ASSET WEIGHTS.

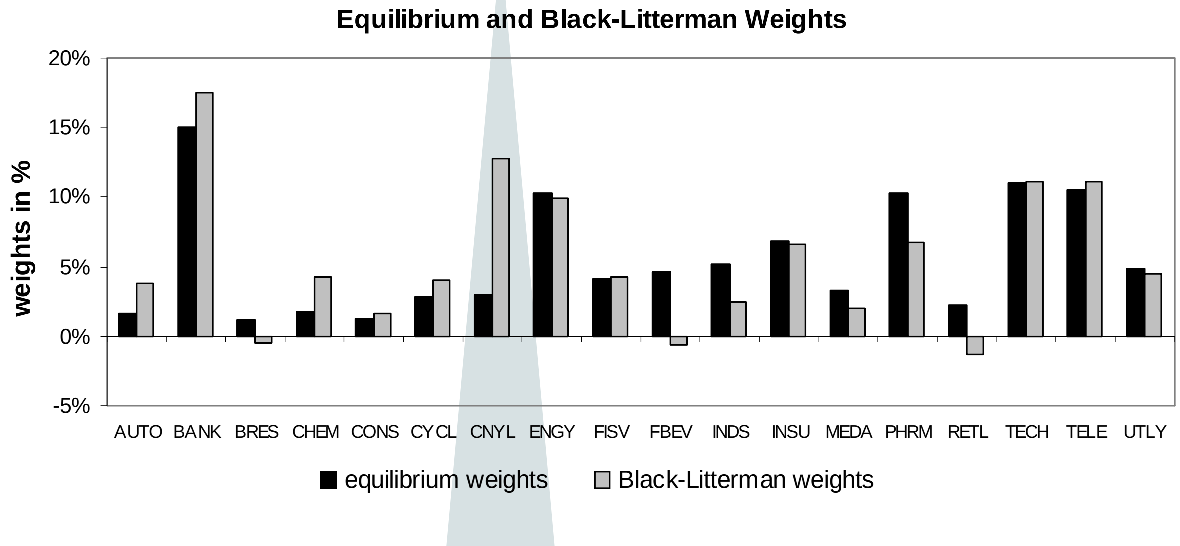
BL compared to straight MV





View on return:

from 6.6% up to 7.5%
(with strong confidence)



Result

- Realistic weight changes when preprocessing with BL
- Volatile weight scenario in straight MV approach

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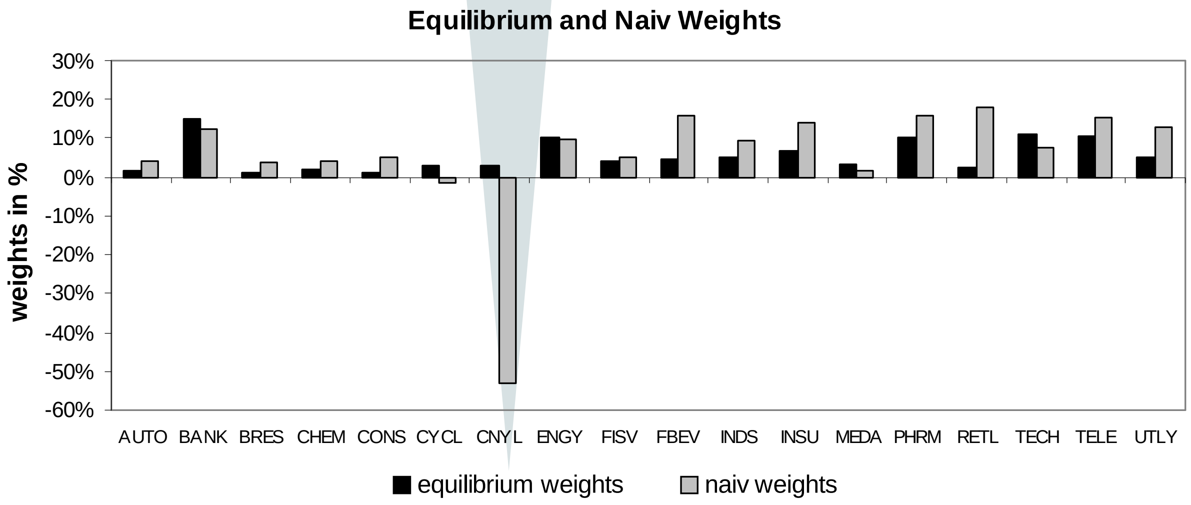
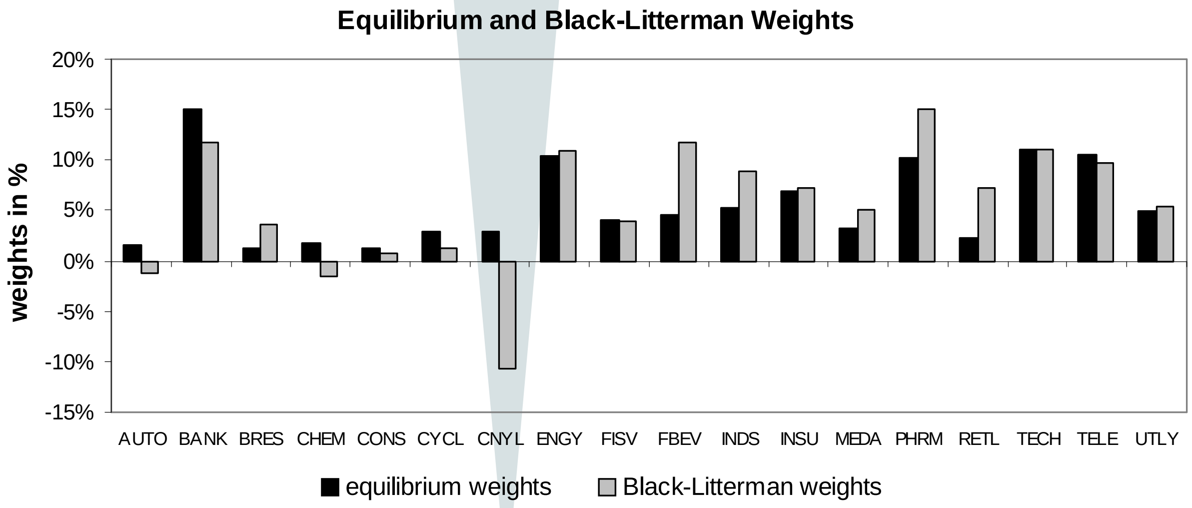
BL compared to straight MV

CNYL



View on return:

from 6.6% down to 5.5%
(with strong confidence)



Result

> Realistic weight changes when preprocessing with BL
> Volatile weight scenario in straight MV approach

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Traditional „Straight MV“ vs „BL + MV“ approach

	straight MV	Black-Litterman + MV
Return estimates:		
	o required for <u>each</u> asset	required only for <u>selected</u> assets
	o assumed as <u>certain</u>	degree of <u>confidence</u>
	o <u>absolute</u> return figures	<u>absolute or relative</u> Views
	o c.p.	consistent
Reference return:		
	o none	equilibrium returns

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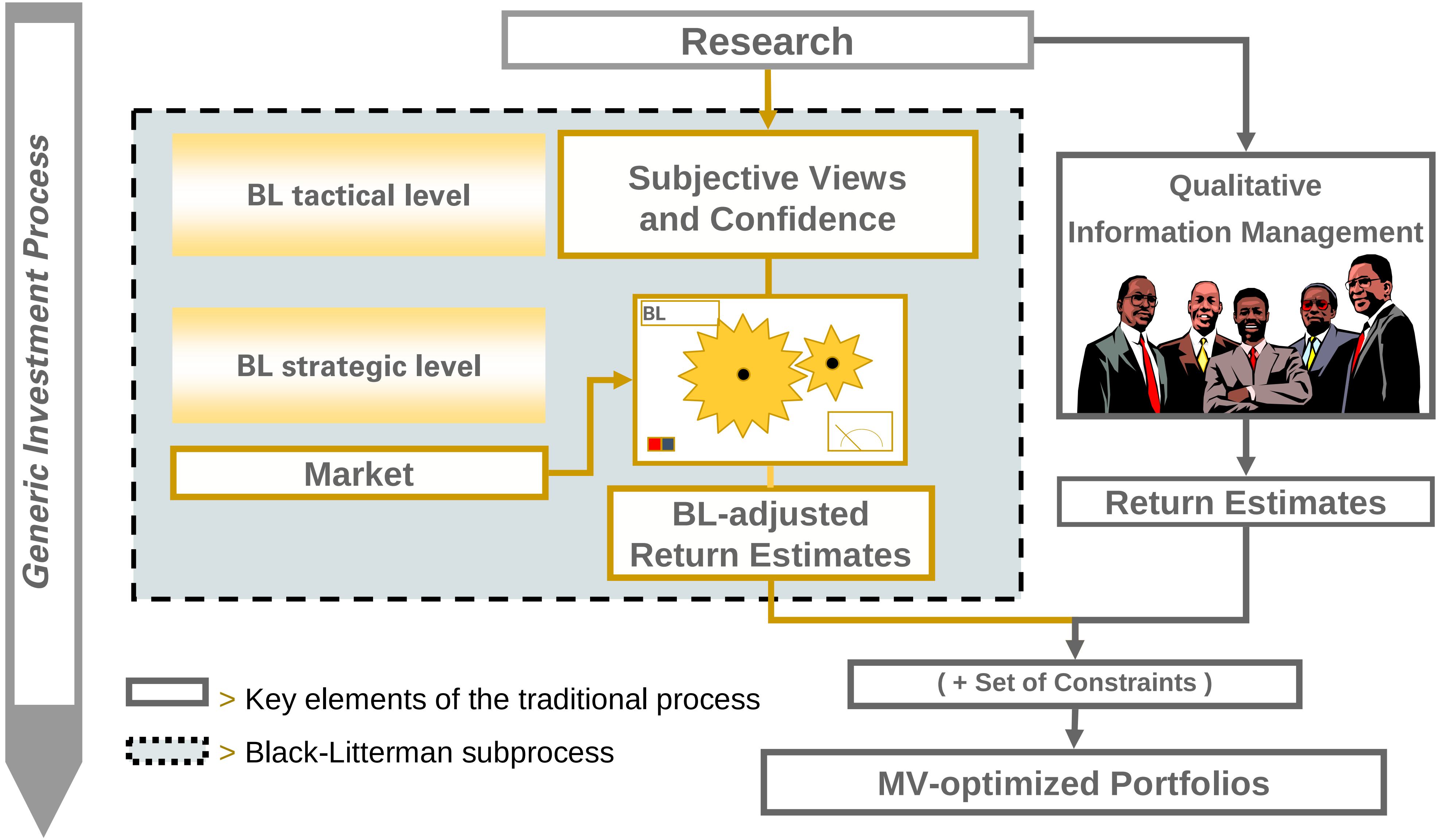
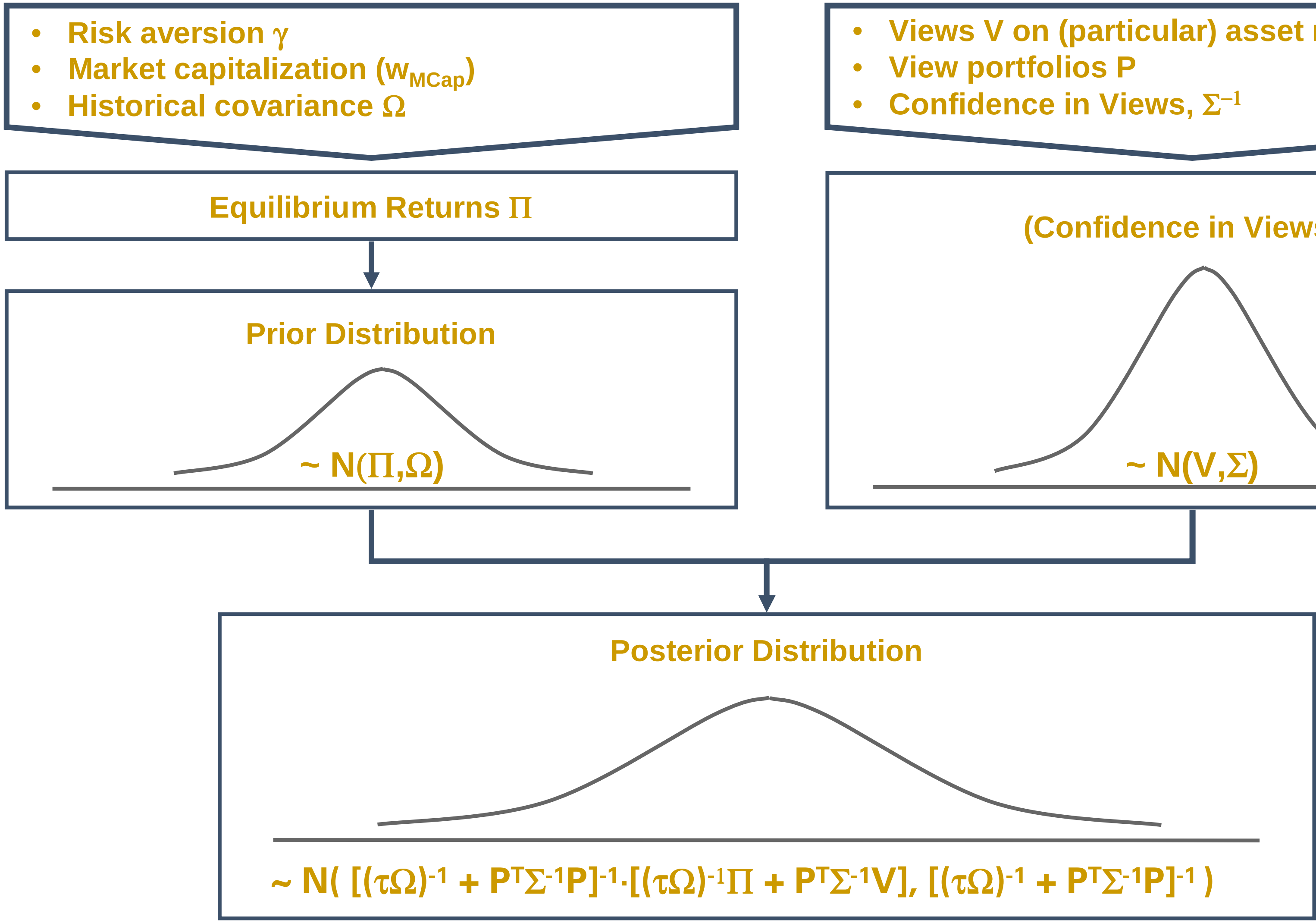
Traditional „Straight MV“ vs „BL + MV“ approach

	straight MV	Black-Litterman + MV
MV-optimized Portfolios:		
	o extreme asset weights	reliable asset weights
	o changes in return estimates	
	$\Rightarrow$ huge weight fluctuations	$\Rightarrow$ moderate weight changes
	o portfolios unreliable	consistent structure „intuitively reasonable“
	o MV-results hardly accepted	higher degree of acceptance
	o reflecting c.p. opinions	„correlated Views“

CONCLUSION 3.
ROLE OF BL IN THE INVESTMENT PROCESS.



THE BAYESIAN VIEWPOINT.



BLACK-LITTERMAN-APPROACH.
(PROBABLY) MORE INSIGHTS...

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BLACK-LITTERMAN - APPENDICES.
MORE FORMAL DETAILS.

- The BL Master Equations
- Interpreting Confidence

BLACK-LITTERMAN - THE FORMULAS:
MASTER EQUATION FOR THE BL-RETURN ESTIMATES.

> Solution in the case of certain estimates ($\Sigma \equiv$ zero matrix):

$$\overline{E}(R) = \Pi + (\tau\,\Omega)\,P^{\mathrm{T}} \cdot \left(P(\tau\,\Omega)\,P^{\mathrm{T}}\right)^{-1} \cdot \left(V - P\,\Pi\right)$$

> Solution in the case of uncertain estimates (Σ = diagonal matrix):

$$\overline{E}(R) = \left[(\tau\,\Omega)^{-1} + P^{\mathrm{T}}\Sigma^{-1}P\right]^{-1} \cdot \left[(\tau\,\Omega)^{-1}\Pi + P^{\mathrm{T}}\Sigma^{-1}V\right]$$

> The constraints $P \cdot E(R) = V + e$ are implicitly fulfilled.

FORMAL PROOF FOR THE „CERTAIN CASE“.

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Proposition: The optimisation problem $[E(R)-\Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E(R)-\Pi] \rightarrow \min_{E(R)}$ s.t. $P \cdot E(R) = V$

yields variance-minimum returns $\overline{E}(R) = \Pi + (\tau\,\Omega)\,P^{\mathrm{T}} \cdot \left(P(\tau\,\Omega)\,P^{\mathrm{T}}\right)^{-1} \cdot \left(V - P\,\Pi\right)$

Proof:

Lagrangian: $L := [E-\Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E-\Pi] - \lambda \cdot (PE - V)$

f.o.c.'s: (1) $\frac{\partial L}{\partial E} = 0$ and (2) $\frac{\partial L}{\partial \lambda} = 0$

"scalarizing": $$\frac{\partial L}{\partial E_i} = \frac{\partial}{\partial E_i}\left\{\tau^{-1}\sum_{j,k} E_j \Omega^{-1}_{jk} E_k - \sum_k \lambda_k \left(\sum_j P_{kj} E_j - V_k\right)\right\} = 2\tau^{-1}\sum_k \Omega^{-1}_{ik} E_k - \sum_k P_{ki}\lambda_k$$

$$\frac{\partial L}{\partial \lambda_i} = -\frac{\partial}{\partial \lambda_i}\sum_k \lambda_k \left(\sum_j P_{kj} E_j - V_k\right) = -\left(\sum_j P_{ij} E_j - V_i\right)$$

"*re*vectorizing": (1') $\frac{\partial L}{\partial E} = 2(\tau\,\Omega)^{-1}\,E - 2(\tau\,\Omega)^{-1}\,\Pi - P\lambda = 0$ and (2') $\frac{\partial L}{\partial \lambda} = PE - V = 0$

Solve Eq. (1') for E, insert in Eq.(2'), thereof expression for λ, result follows with Eq. (1'). ◊

BLACK-LITTERMAN - THE FORMULAS - MORE MATH 2: FORMAL PROOF FOR THE „UNCERTAIN CASE“.

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Proposition: The opt. problem $[E(R)-\Pi]^{T}\cdot(\tau\,\Omega)^{-1}\cdot[E(R)-\Pi]\rightarrow\min_{E(R)}$ s.t. $P\cdot E(R)=V+e$

yields variance-minimum returns $\overline{E}(R)=\left[(\tau\,\Omega)^{-1}+P^{T}\Sigma^{-1}P\right]^{-1}\cdot\left[(\tau\,\Omega)^{-1}\Pi+P^{T}\Sigma^{-1}V\right]$

Proof:

$$\text{Given}: \Pi=E(R)+\nu \quad \text{and} \quad V=P\cdot E(R)+e$$

$$\text{Setting} \quad Y:=\begin{pmatrix}\Pi\\ V\end{pmatrix},\quad X:=\begin{pmatrix}I\\ P^{T}\end{pmatrix},\quad W:=\begin{pmatrix}\tau\Omega & 0\\ 0 & \Sigma\end{pmatrix} \quad \text{and} \quad u\sim N(0,W)$$

$$\text{so that} \quad Y=X\cdot E(R)+u \quad \text{and using generalized least square} \quad E(R)=\left(X^{T}W^{-1}X\right)^{-1}X^{T}W^{-1}Y \quad \text{we get}$$

$$E(R)=\left[\begin{pmatrix}I & P^{T}\end{pmatrix}\begin{pmatrix}\tau\Omega & 0\\ 0 & \Sigma\end{pmatrix}^{-1}\begin{pmatrix}I\\ P\end{pmatrix}\right]^{-1}\times\left[\begin{pmatrix}I & P^{T}\end{pmatrix}\begin{pmatrix}\tau\Omega & 0\\ 0 & \Sigma\end{pmatrix}^{-1}\begin{pmatrix}\Pi\\ V\end{pmatrix}\right]$$

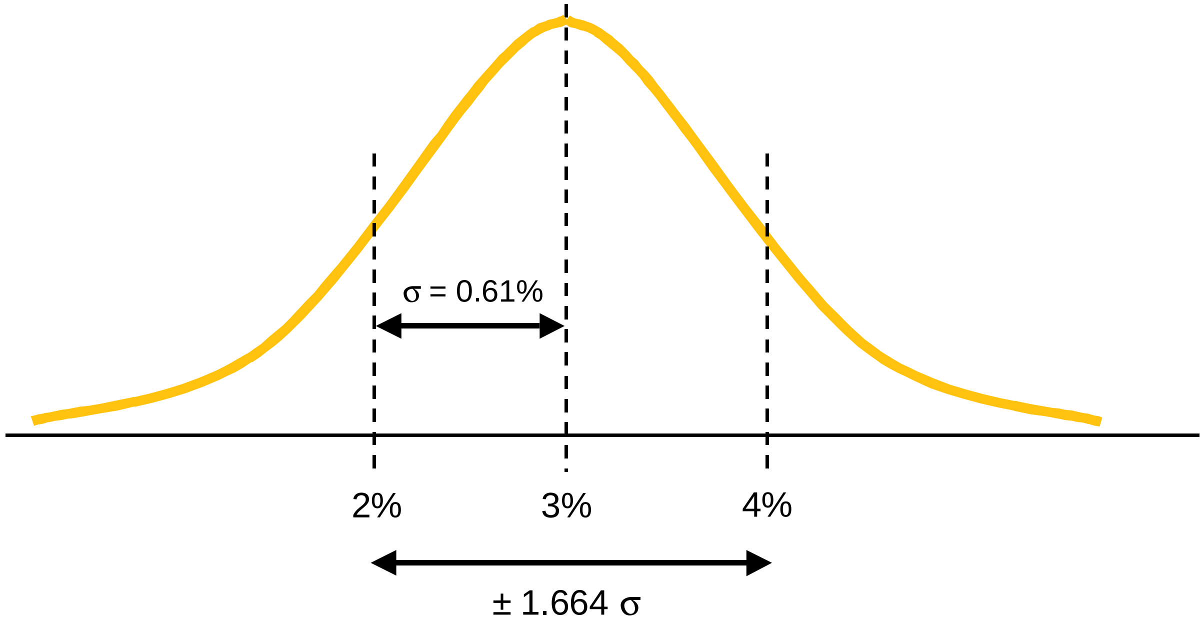
$$=\left[\begin{pmatrix}(\tau\Omega)^{-1} & P^{T}\Sigma^{-1}\end{pmatrix}\begin{pmatrix}I\\ P\end{pmatrix}\right]^{-1}\times\left[\begin{pmatrix}(\tau\Omega)^{-1} & P^{T}\Sigma^{-1}\end{pmatrix}\begin{pmatrix}\Pi\\ V\end{pmatrix}\right]=\left[(\tau\Omega)^{-1}+P^{T}\Sigma^{-1}P\right]^{-1}\times\left[(\tau\Omega)^{-1}\Pi+P^{T}\Sigma^{-1}V\right]\ \Diamond$$

Non-Bayesian proof, taken from *„Asset Allocation Model“, Daniel Blamont, Global Markets Research, Dt.Bank, July 30 2003*

For Bayesian proof see, *e.g., Satchell and Scowcroft* or *Fusai and Meucci*

Technical note on Confidence

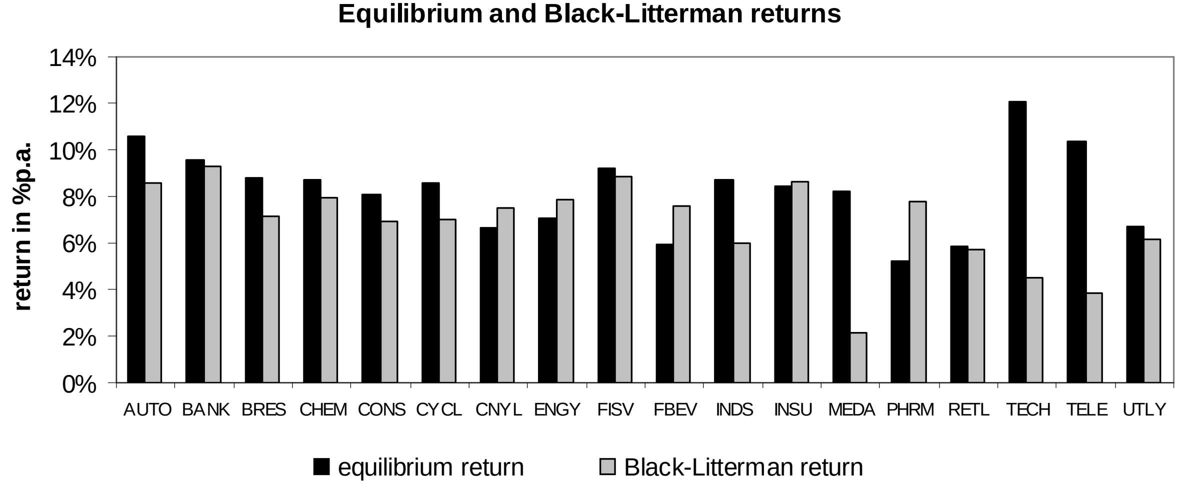
- Comment on determination of *e* : The fact that the amount of, e.g., *relative* outperformance (View 1) of 3% ±1% is assigned a 90% probability is interpreted within a normal distribution.
- *mean* = 3% and *variance* = $VAR = \sigma^2 = (0.61\%)^2 = e_1 \equiv \Sigma_{11}$.

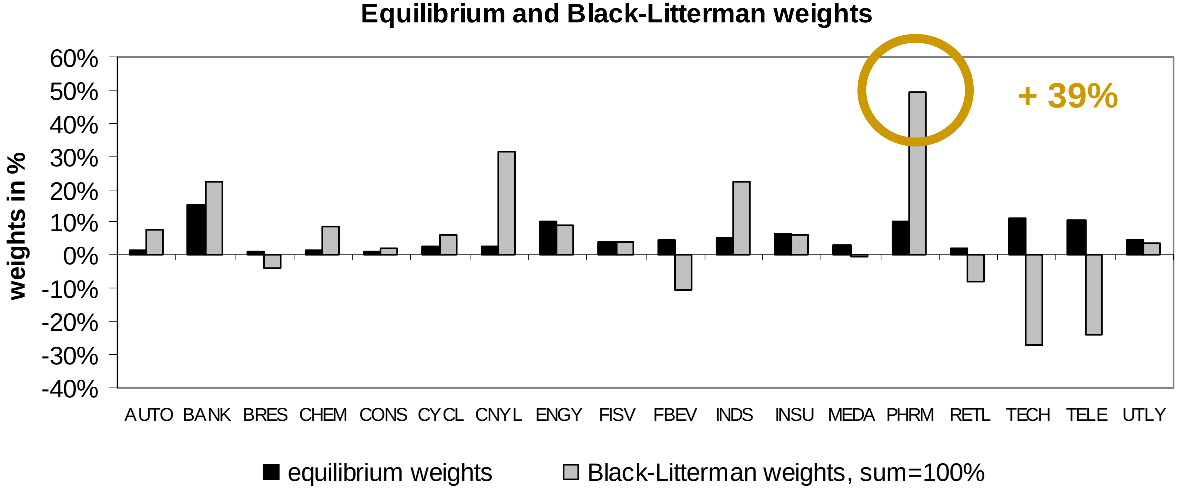


BLACK-LITTERMAN - CONFIDENCE.
SENSITIVITY OF BL-RETURNS AND BL-WEIGHTS ON DIFFERENT LEVELS OF CONFIDENCE.

> **Strong confidence**

> Large changes in weights due to the “strong views“ (as expected)





BLACK-LITTERMAN - CONFIDENCE.
SENSITIVITY OF BL-RETURNS AND BL-WEIGHTS ON DIFFERENT LEVELS OF CONFIDENCE.

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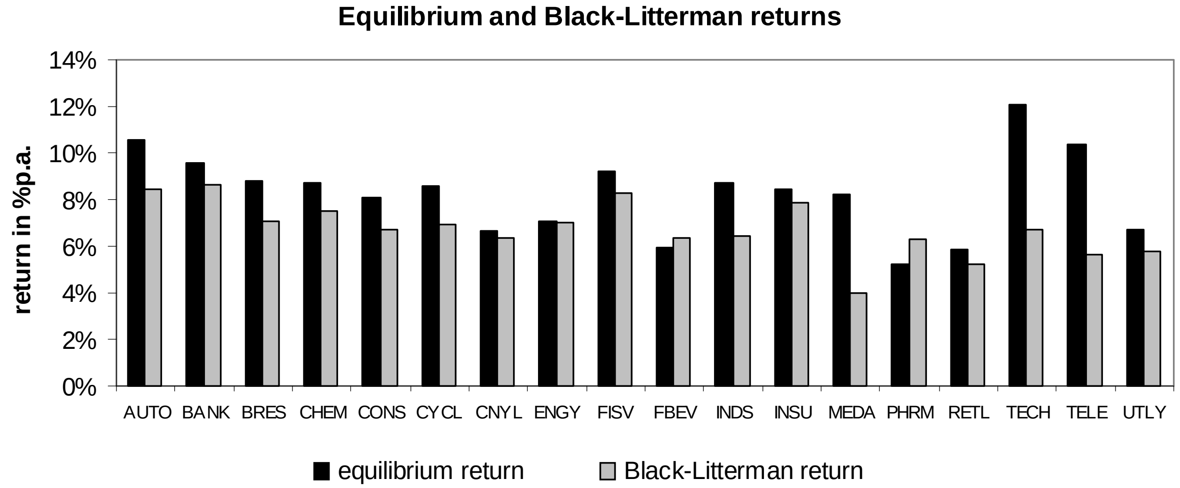
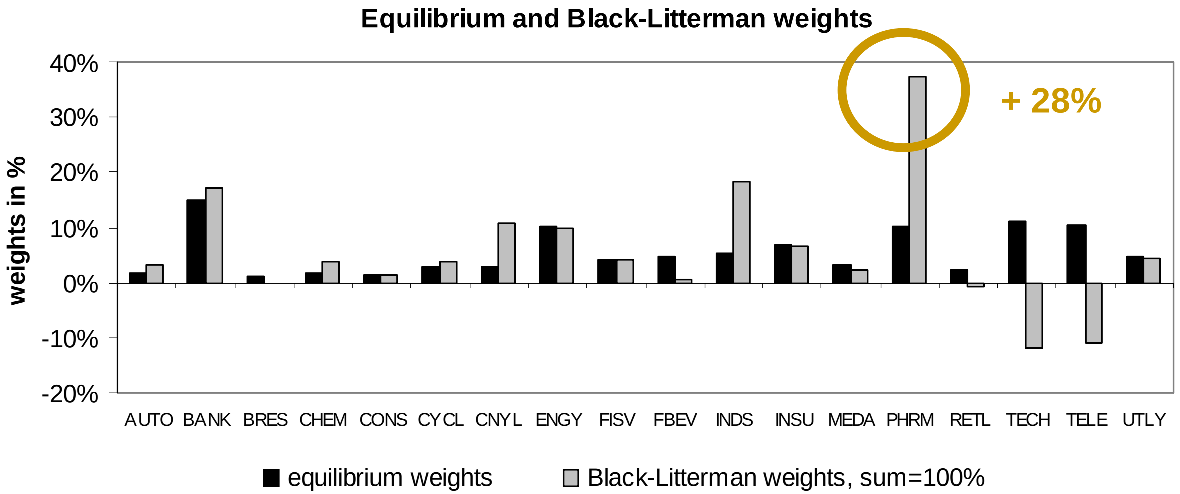
> **Mid confidence**

> Moderate changes in weights (as expected)

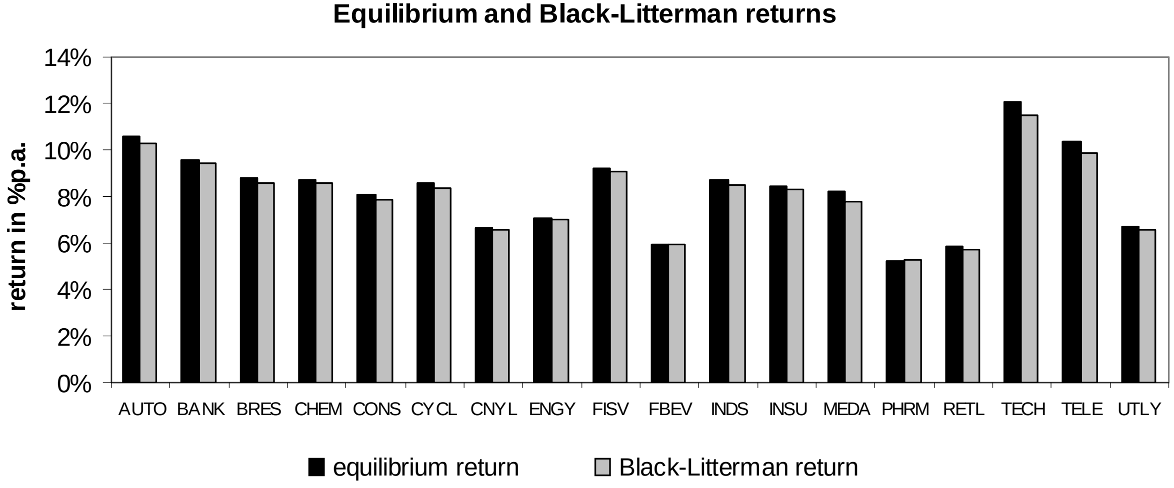


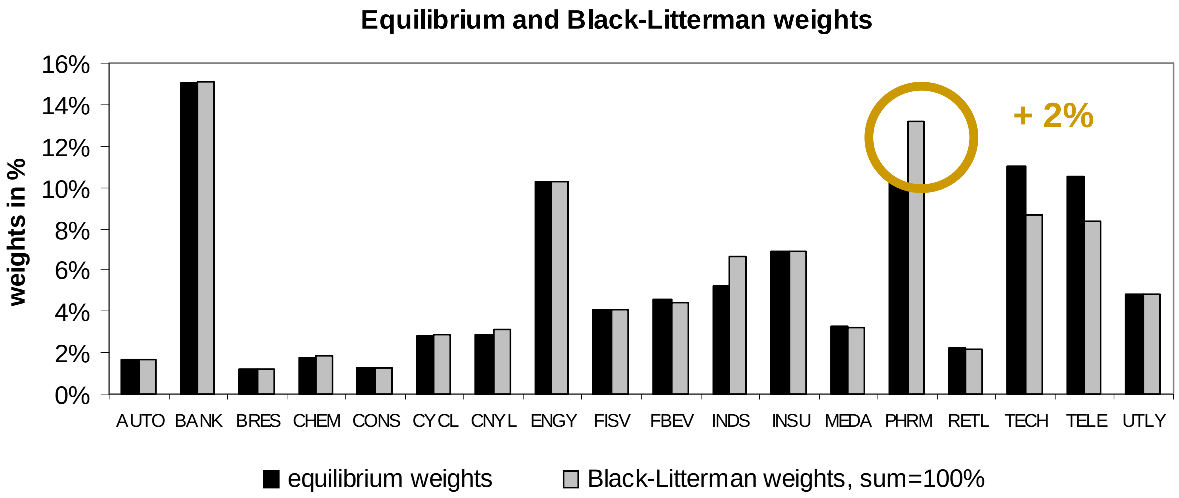


BLACK-LITTERMAN - CONFIDENCE.
SENSITIVITY OF BL-RETURNS AND BL-WEIGHTS ON DIFFERENT LEVELS OF CONFIDENCE.

> **Poor confidence**

> Small changes in weights (as expected)





THANK YOU
FOR YOUR ATTENTION.

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