A Structural Model for Electricity Prices with Spikes

Measurement of Jump Risk and Optimal Policies for Hydropower Plant Operation

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ABSTRACT

This paper proposes a new, structural model for electricity prices. We show that unlike other electricity price models, such as the jump diffusion model and the Box-Cox transformation model, the structural model can directly and accurately incorporate the relationship between electricity demand and price spikes. We also illustrate the usefulness of the structural model for optimal power generation and risk management using the example of a pump-storage hydropower plant. The structural model can describe the probability of price spikes easily in terms of electricity demand, and provides more realistic optimal operation policies than the jump diffusion model.

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1. Introduction

The deregulation of electricity markets has caused electricity prices to spike (i.e., to increase suddenly and drastically). The risk of price spikes affects power companies in both positive and negative ways. On one hand, price spikes may provide profitable opportunities to companies if they can sell electricity at high "spiked" prices in spot markets. On the other hand, the price spikes may be a burden if the companies have contracts to supply electricity at low, predetermined prices. Power companies are now required to manage such risks.

For the purpose of risk management, two types of model have been developed so far to describe price spikes. One is the jump diffusion-type model (e.g., Johnson and Barz (1999), Deng (2000)) that formulates electricity prices as jump diffusion processes [1]. Jump diffusion models are successful at generating relatively easy pricing formulae for electricity derivatives. However, to keep the calculation simple, they focus only on prices and ignore the relationship between demand/supply and prices. Consequently, jump diffusion models do not capture the fact that the price tends to spike in summer (and sometimes in winter) because electricity demand is high and sometimes hits supply capacity.

The other type of model tries to relate electricity prices to supply and demand. The idea behind these models is simple. Demand and supply determine price: these models just need to describe this relationship. Skantze, Gubina, and Ilic (2000) formulates the electricity demand using a mean-reverting stochastic process and describes price as the exponential function of demand. Barlow (2002) employs the inverse function of the Box-Cox transformation, instead of the exponential function. These models are on the right track to accommodate the economic background of the electricity market more carefully than naïve jump diffusion models. However, their results are not completely satisfactory. For example, the former cannot generate price spikes large enough to match actual data owing to the small curvature of the exponential function. The latter improves the size of the price spikes with the inverse Box-Cox function, but its curvature is still too small to capture the drastic price increase. Moreover, it does not use actual demand data in its analysis, thereby obscuring the importance of the seasonality of electricity demand in describing price spikes, although Barlow (2002) mentions incorporating the demand seasonality as one important extension of the model.

This paper shares the same theoretical idea as these preceeding papers, but extends the model to explicitly incorporate the actual relationship between the electricity price and supply/demand. We call this model a structural model for electricity prices. We estimate the structural model using actual demand and price data from the PJM electricity market[2] and show that it can exhibit a more realistic demand-price relationship than the preceeding models at least for the PJM market. We also show that the seasonality of demand is an important factor for describing the price spikes accurately in the inverse Box-Cox model. Moreover, we show that the structural model can describe the timing and size of price spikes more accurately than the other models.

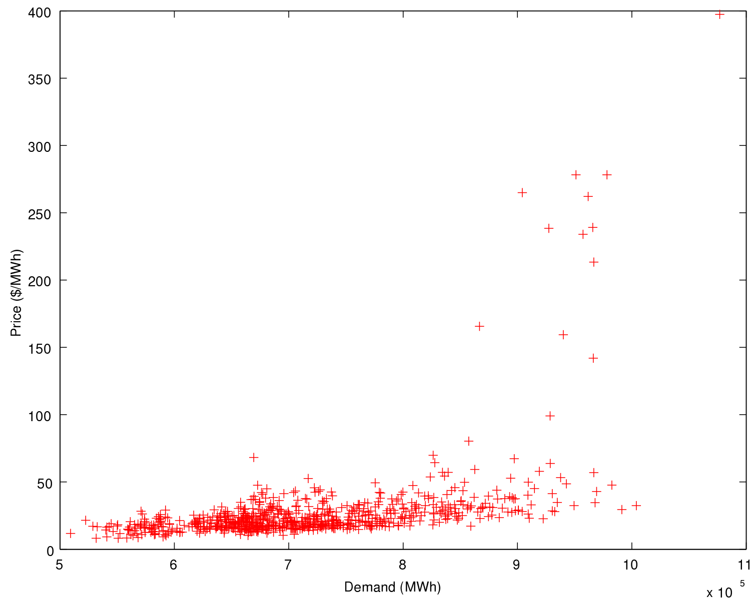

Figure 1. Electricity Prices and Demand (=Supply)

The idea behind the structural model is based on a simple observation of the electricity market. Figure 1 plots electricity demand (=supply) against price in the PJM electricity market from January 1, 1999 to December 31, 2000. Since electricity demand is inelastic to price in

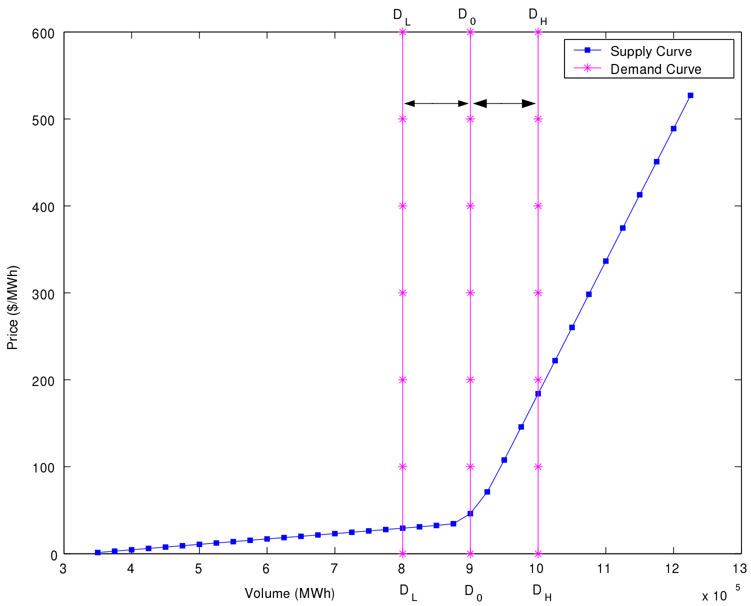

Figure 2. Supply-Demand Curve

the short run, the demand curve is depicted as a vertical line. The supply curve can be depicted as in Figure 2. The supply curve has this shape because in the short run, the number of power generating facilities is almost fixed, and because power companies have to start operating more costly facilities beyond the threshold level of supply. As a result, the slope of the supply curve suddenly becomes steeper.

The demand curve, which is a vertical line in the short run, stochastically fluctuates in parallel, as in Figure 2. When the demand is in the region where the slope of the supply curve is flat (the left side of D_0D_0), demand fluctuation does not affect the price much. But when it enters the region where the slope is steep (the right side of D_0D_0), the price increases suddenly and drastically. This is how an increase in demand causes a price spike. It is also easy to see that the price tends to spike when the demand level is high, which explains why price spikes tend to occur in summer (and sometimes in winter).

To incorporate the relationship between demand (= supply) and price explicitly, it is important to specify the supply and demand curves. Like Skantze, Gubina, and Ilic (2000) and

Barlow (2002), we assume that short-run electricity demand is inelastic to price and thus describe it using a stochastically moving vertical line, as in Figure 2. However, unlike Skantze, Gubina, and Ilic (2000), which employs the exponential function, or Barlow (2002), which employs the inverse Box-Cox transformation, we assume that the supply curve has a hockey stick shape and consists of two lines - one flat and the other steep - linked by a quadratic curve, as in Figure 2. We estimate the supply curve using the hockey stick regression. Note that the intersection of the demand and supply curves gives the equilibrium electricity price. The hockey stick shape of the supply curve allows the structural model to generate sudden and large price changes more easily than the models with the exponential supply function or the inverse Box-Cox transformation supply function.

In the following, we first estimate the models using the data from the PJM electricity market. We then use the estimated models and show by simulation that the structural model can capture the characteristics of the price spikes - especially their timing - more accurately than the jump diffusion models and the Box-Cox transformation model. We also point out the importance of demand seasonality in describing the price spikes not only for the structural model, but also for the Box-Cox transformation model. Finally, we compare the optimal operation strategies for a pump-storage hydropower generator using the structural model and the advanced jump diffusion model of Thompson, Davison, and Rasmussen (2003). The result shows that the structural model can provide realistic optimal operation strategies based on demand levels in a much simpler way than the jump diffusion model.

The paper is organized as follows: Section 2 formulates the structural model. Section 3 compares the structural model with other models. Section 4 examines the optimal operation strategies for a pump-storage facility using the structural model and the jump diffusion model. Section 5 offers concluding remarks and sketch directions for future research.

2. The Structural Model

2.1. Demand Curve

Casual observation tells us that electricity demand is on average high in summer and even higher on hotter days. This leads us to decompose electricity demand into two parts: the seasonality part that represents the seasonal trend of demand and the fluctuation part that changes stochastically day by day. Thus, we formulate electricity demand using the sum of the deterministic seasonal term and the stochastic daily term in the following way. We denote by D_t and $\bar{D}_t$ daily electricity demand and its average for several years on the same day, respectively. X_t is calculated using the deviation of D_t from $\bar{D}_t$ as

$$X_t = D_t - \bar{D}_t. \tag{1}$$

We assume that X_t follows an Ornstein-Uhlenbeck process

$$dX_t = (\mu_X - \lambda_X X_t)dt + \sigma_X dw_{1t}. \tag{2}$$

The mean-reversion of X_t implies that when demand deviates from the average, it tends to return to the average. Note that electricity demand is strongly affected by the temperature. The mean-reversion property of demand is a reflection of that of the temperature.

Since electricity demand is inelastic to price in the short term, we suppose the demand curve to be independent of price and paralleled to the price axis. In this formulation, the demand curve shifts from side to side as it fluctuates stochastically (Figure 2).

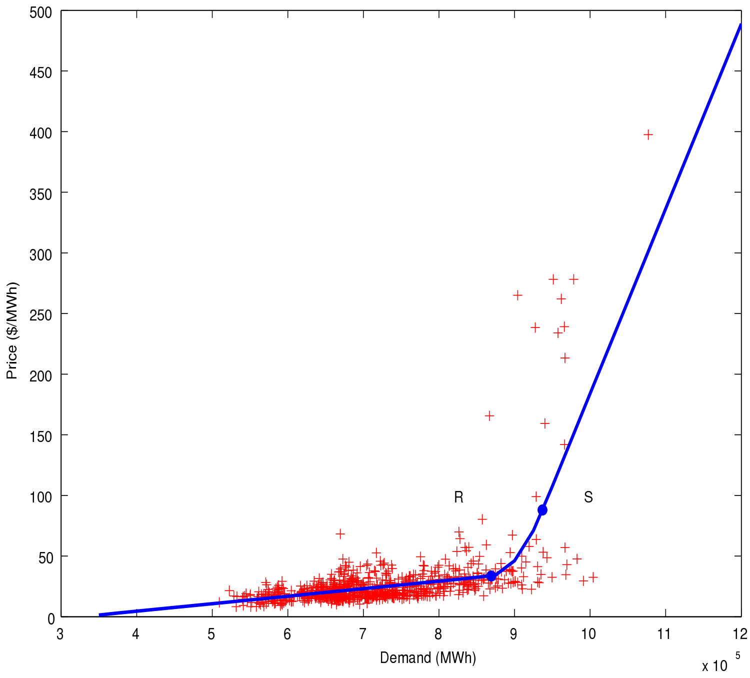

Figure 3. Hockey Stick Regression Model

2.2. Supply Curve

Power companies start operating their generating facilities with the lowest marginal costs. As the electricity supply increases, they utilize these facilities with higher marginal costs to generate more electricity. The historical data from the PJM market from January 1, 1999 to December 31, 2000 are illustrated in Figure 3. The figure shows that the electricity price increases as the supply increases.

Figure 3 exhibits the most significant characteristics of the supply curve i.e., the slope. Although the slope of the supply curve is flat until 900,000MWh, it suddenly becomes steep above 900,000MWh. This is because when the standard facilities cannot supply sufficient electricity, other power plants with high marginal costs must be put into operation to keep up with demand. This occurs when demand exceeds about 900,000MWh. As a result, the electricity price suddenly increases and exhibits spikes.

To describe such price spikes accurately, we introduce a supply curve with a hockey stick shape that incorporates the two different slope lines and connects them with a quadratic curve. At the two end points, the tangential line of the curve is equal to the slope of the connecting line. The model is shown as the solid line in Figure 3. The model requires that we estimate the two slopes of β_1 and β_2, the intercept on the left-hand side line of α_1, and the two connecting points of $z-s$ and $z+s$. z denotes the middle point of the domain of the quadratic curve. The connecting points deviate from the middle point with the distance of $\pm s$ (Figure 3).

In sum, the electricity supply curve with price (P_t) and supply (S_t) is given by the following functions

$$P_t = f(S_t) = \alpha_1 + \beta_1 S_t + \varepsilon_t \qquad (S_t \leq z-s) \tag{3}$$

$$P_t = f(S_t) = a + bS_t + cS_t^2 + \varepsilon_t \qquad (z-s < S_t < z+s) \tag{4}$$

$$P_t = f(S_t) = \alpha_2 + \beta_2 S_t + \varepsilon_t \qquad (S_t \geq z+s) \tag{5}$$

where

$$x_1 = z-s, \qquad x_2 = z+s, \qquad a = \alpha_1 + \beta_1 x_1 - bx_1 - cx_1^2$$
$$b = \frac{x_2\beta_1 - x_1\beta_2}{x_2 - x_1}, \qquad c = \frac{\beta_2 - b}{2x_2}, \qquad \alpha_2 = -\beta_2 x_2 + a + bx_2 + cx_2^2.$$

2.3. Equilibrium Price

In the structural model, the electricity price is obtained as the equilibrium price of supply and demand. Since demand is inelastic to price, the equilibrium price is given from the supply curve by setting supply equal to demand. That is, we set $S_t = D_t$ on the supply curve and obtain the equilibrium price $P_t = f(S_t) = f(D_t)$. From equation (3) to (5), we have

$$P_t = f(D_t) \tag{6}$$

$$D_t = X_t + \bar{D}_t \tag{7}$$

$$dX_t = (\mu_X - \lambda_X X_t)dt + \sigma_X dw_{1t}. \tag{8}$$

3. Empirical Analysis of Electricity Price

3.1. Data

We use daily average price and daily demand data calculated from the hourly price and demand of the PJM electricity market. We do not employ hourly data because our aim is not to analyze high frequency data. We use the prices at the PJM Western Hub as a proxy for the whole PJM market. The data length is from April 1, 1998 to March 31, 2002.

The basic statistics are presented in Table 1. Here, we can find the large standard deviation,

	Electricity Price ($\$/MWh$)	Electricity Volume (MWh)
Average	26.8	714770.0
Standard Deviation	25.0	95596.4
Standard Error	0.65	2501.0
Sample Number	1461	1461
Variance	624.2	9.1×10^9
Skewness	8.0	0.7
Kurtosis	80.8	0.5

Table 1
Basic Statistics of Electricity Price and Volume

skewness, and kurtosis of the electricity prices. In comparison, the distribution of the volumes has a smaller skewness and kurtosis than that of the prices. The distribution of the volumes is closer to a normal distribution than that of the prices.

3.2. Parameter Estimation

3.2.1. Estimation of the Demand Process

To estimate demand, we model the deviation X_t of the electricity demand from its average by an AR(1) process,

$$\Delta X_t = \alpha_0 + \alpha_1 X_t + \nu_t \tag{9}$$

where $\nu_t \sim N(0, \sigma_\nu^2)$ is disturbance. We denote the set of parameters by $\Theta = (\alpha_0, \alpha_1, \sigma_\nu^2)$. We estimate them by a maximum likelihood method with the initial values obtained by a least square method. We present the estimates in Table 2. Both parameters of α_1 and σ_ν^2 are

Parameter	α_0	α_1	σ_ν^2
Estimate	-19.40	-0.33	$2.32 * 10^9$
t-statistic	-0.02	-37.72	28.85
Log-likelihood	-17813.83		
SIC	35633.67		
AIC	35649.52		

Table 2
Parameter Estimates for PJM Demand

significant in *t*-statistics, while α_0 is not significant. This result reveals that the mean-reverting property (represented by α_1) is comparatively strong.

We transform these estimates in a discrete time model to those in a continuous time model by the following transformation (Clewlow and Strickland (2000)). The parameters (μ_X, λ_X, σ_X) in equation (14) are given by

$$\lambda_X = -\log(1+\alpha_1), \quad \mu_X = \frac{\alpha_0}{\alpha_1}\log(1+\alpha_1), \quad \sigma_X = \sigma_\nu \sqrt{\frac{2\log(1+\alpha_1)}{(1+\alpha_1)^2 - 1}}$$

where σ_ν is the standard deviation of the errors ν_t in equation (9). From Table 2, we obtain

$$\lambda_X = 0.40, \quad \mu_X = 0, \quad \sigma_X = 58139.83.$$

3.2.2. Estimation of Supply Curve Parameters

As in Figure 3, the supply curve has different slopes below and above 900,000MWh. This reflects the constitution of power plants. Low marginal cost plants, such as nuclear and coal-fired plants, are operated when the supply is low, while the high marginal cost plants, such as oil-fired plants and other resources, are operated when the supply is high. We employ the hockey stick shaped model to represent such characteristics of the supply curve. We estimate this model using hockey stick regression. The data are the PJM daily average price and daily demand from January 1, 1999 to December 31, 2000. We use the data with the shorter period of two years to keep the constitution of power plants fixed. We use a nonlinear least square method to estimate the parameters. The result is presented in Table 3. Judging from the

Parameter	α_1	β_1	β_2	z	s
Estimate	-20.31	$6.21 * 10^{-5}$	$1.53 * 10^{-3}$	$9.03 * 10^5$	$3.43 * 10^4$
t-statistic	-2.68	5.77	10.06	124.55	1.89
Log-likelihood	-3292.03				
SIC	6617.03				
AIC	6594.06				

Table 3
Estimation of Supply Curve Parameters

t-statistics, all parameters except the parameter s are statistically significant.

In equilibrium, supply equals demand. Thus, the hockey stick regression model shows that the price change is small in the low-demand region and large in the high-demand region. For example, an increase in demand (i.e., supply) from 700,000MWh to 800,000MWh increases price by only \$6/MWh. In contrast, an increase in demand from 1,000,000MWh to 1,100,000MWh increases the corresponding price dramatically, by \$153/MWh.

3.3. Comparison with Other Models

We compare the structural model with two existing models: a jump diffusion model and a Box-Cox transformation model.

3.3.1. Existing Models

Jump Diffusion Model

As a jump diffusion model, we consider the geometric mean-reverting process with jumps in which the logarithm of the price is mean-reverting and has an independent Poisson jump whose jump size is normally distributed. The daily average electricity price, denote by P_t is defined by the following stochastic differential equation :

$$d \log P_t = \alpha(\beta - \log P_t)dt + vdw + Jd\pi(h), \qquad P_0 = P_0 \tag{10}$$

where v is constant, π is a Poisson process with the parameter h representing the number of jumps per annum, and $J \sim N(\mu_J, \sigma_J^2)$ representing the jump size. This is one of the jump diffusion models of electricity prices analyzed by Johnson and Barz (1999). As in Johnson and Barz (1999), w, J, and π are usually assumed to be independent. This is for calculation ease. Most jump diffusion models are aimed at pricing derivatives written on electricity prices and the assumption of independence makes this task relatively easy. However, because of this assumption, the jump diffusion models cannot capture the relationship between electricity demand and price, and hence cannot incorporate the timing of jump occurrences.

Box-Cox Transformation Model

Barlow (2002) proposes a Box-Cox transformation model in which the electricity price P_t is transformed into Y_t, the normalized demand for electricity with a Box-Cox transformation. Y_t is assumed to follow an Ornstein-Uhlenbeck process

$$P_t \equiv f_\alpha(Y_t) = \begin{cases} (1+\alpha Y_t)^{\frac{1}{\alpha}} & \alpha \neq 0 \\ e^{Y_t} & \alpha = 0 \end{cases} \tag{11}$$

$$dY_t = (\mu_B - \lambda_B Y_t)dt + \sigma_B dw_t \tag{12}$$

where the inverse function of $f_\alpha(Y_t)$ is the Box-Cox transformation.

$$f_\alpha^{-1}(x) \equiv g_\alpha(x) = \begin{cases} \frac{x^\alpha - 1}{\alpha} & \alpha \neq 0, \quad x > 0 \\ \log x & \alpha = 0, \quad x > 0 \end{cases}$$

In this model, as normalized demand Y_t increases, the corresponding price also increases. In this sense, it captures the relationship between the electricity demand and price.

Note that Barlow (2002) also suggests the possibility of incorporating seasonality in the variable Y_t as a possible extension. However in the actual analysis, Barlow (2002) does not indeed incorporate seasonality in the variable Y_t.

Extended Box-Cox Transformation Model with Seasonality

To study the importance of demand seasonality in predicting price spikes, we extend a Box-Cox transformation model to incorporate seasonality. For this purpose, we model electricity demand, as in the structural model, using equations (13) and (14) where $\bar{D}_t$ represents the demand seasonality, or the average demand, and X_t represents the deviation. Then, we model the relationship between price and demand using the Box-Cox transformation.

$$D_t = X_t + \bar{D}_t \tag{13}$$

$$dX_t = (\mu_X - \lambda_X X_t)dt + \sigma_X dw_{1t} \tag{14}$$

$$P_t \equiv f_\alpha(D_t) = \begin{cases} (1+\alpha\frac{D_t - b}{c})^{\frac{1}{\alpha}} & \alpha \neq 0 \\ e^{\frac{D_t - b}{c}} & \alpha = 0 \end{cases} \tag{15}$$

3.3.2. Comparison of Price Spikes

Like Das (2002), we estimate the parameters of the jump diffusion model using a maximum likelihood method, assuming that in the time interval Δt the jump occurs once or not at all. We use the same data for the estimation of these models as for that of the structural model. We estimate the parameters of α, β, and v^2 of the mean-reverting process and of q, μ_J, and σ_J of the Poisson jump process using the discrete AR(1) model

$$\Delta \log p_t = \alpha(\beta - \log p_t)\Delta t + v\Delta z + J(\mu_J, \sigma_J^2)\Delta\pi(q) \tag{16}$$

of equation (10). Here, q denotes the jump probability. μ_J and σ_J denote the mean and standard deviation of the jump size. We set $\Delta t = \frac{1}{365}$. $\Delta\pi(q)$ is the increment of the discrete time Poisson process (Das (2002)) and is approximated by the Bernoulli distribution whose parameter is $q = h\Delta t + O(\Delta t)$. The estimation methods and the results are given in Appendix A.

We follow Barlow (2002) in order to estimate the parameters of a Box-Cox transformation model. The data is the same as that used in the structural model. We assume that the variable Y_t transformed from the price P_t with the Box-Cox transformation is governed by an AR(1) process

$$Y_k = b + \rho Y_{k-1} + \theta^{\frac{1}{2}}\eta_k, \qquad 0 \leq k \leq n \tag{17}$$

where $\eta_k \sim i.i.d.N(0,1)$. We estimate the parameters of α in equation (11) and of b, ρ, and θ in equation (17). The estimation methods and the results are given in Appendix A.

We also estimate the parameters of the extended Box-Cox transformation model with the demand seasonality. We employ the Box-Cox inverse transformation function to represent the supply curve. Again, the data is the same as in the structural model. The model is expressed as

$$P_t = f_\alpha(D_t) = (1 + \alpha\frac{D_t - b}{c})^{\frac{1}{\alpha}} + \varepsilon_t. \tag{18}$$

We use a nonlinear least square method to estimate the parameters (α, b, and c in equation (18). The estimation methods and the results are given in Appendix A.

We implement the simulations of the jump diffusion model, the Box-Cox transformation model (without demand seasonality), and the extended Box-Cox transformation model with demand seasonality by using the above estimates. We generate the sample paths of four years' worth of prices using the models 50 times. We select one of the electricity price paths from the four-year period and compare it with that of the structural model. The historical data is also presented in Figure 4.

Figure 4 shows that the jump diffusion model and the Box-Cox transformation model generate random spikes and do not fit the historical data well. The extended Box-Cox transformation model with demand seasonality shows much improvement. It generates the spikes in the same time intervals as the historical data. This result suggests the importance of demand seasonality in describing price spikes based on supply and demand.

However, close inspection shows that the extended Box-Cox transformation model with demand seasonality exhibits a greater number of small size spikes than the historical data, say, about $\$50/MWh$. This is because the curvature of the Box-Cox transformation function is too small to accommodate the sudden change in slope of the actual electricity supply curve. In contrast, the structural model exhibits spikes in the same time intervals and of almost the same size as those in historical data. This is because it uses the hockey stick regression model to capture the sudden slope change in the supply curve more accurately. This result suggests that the structural model describes the historical data better than the other three models: the jump diffusion model, the Box-Cox transformation model, and even the extended Box-Cox transformation model with demand seasonality.

We also simulate the electricity prices with these models by generating 500 years' worth of sample paths and compare the number of spikes in each season [3]. The result is presented in Table 4. We show the number of spikes [4] for four years in order to correspond to the length of the historical data where we define prices above $\$70/MWh$ as the price spikes.

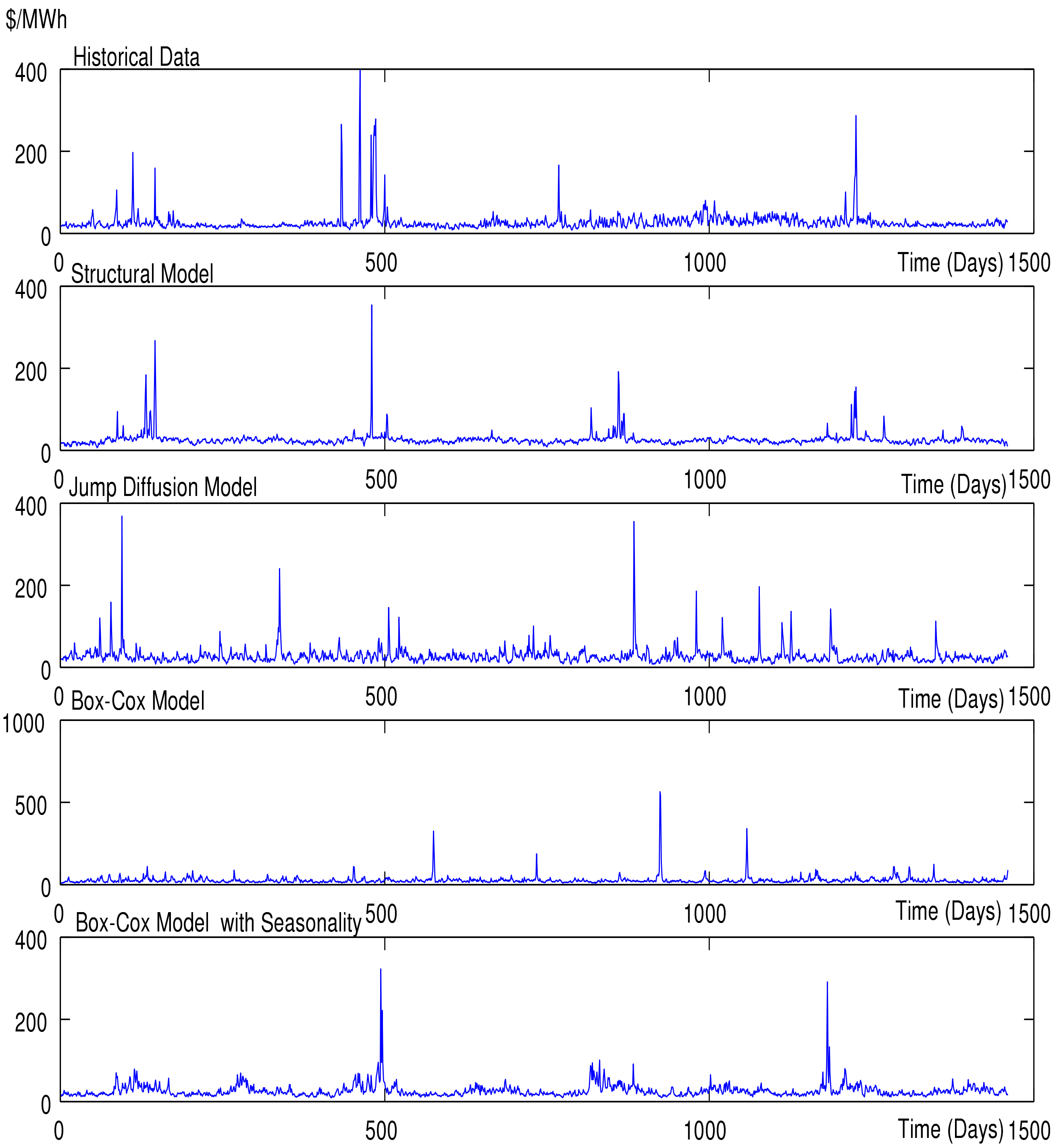

Figure 4. Comparison of Electricity Prices

	Spring	Summer	Autumn	Winter
Historical Data	1	22	0	2
Structural Model	0	23	0	2
Jump Diffusion Model	7	8	8	8
Box-Cox Model	6	7	7	6
Extended Box-Cox Model with Seasonality	0	20	0	1

Table 4
The Number of Spikes in Every Season

Table 4 shows that the structural model and the Box-Cox transformation model with demand seasonality generate more spikes in summer when electricity demand increases, while the jump diffusion model and the Box-Cox transformation model generate spikes independently of the seasons. This result again shows the importance of demand seasonality in accurately modeling electricity prices and price spikes.

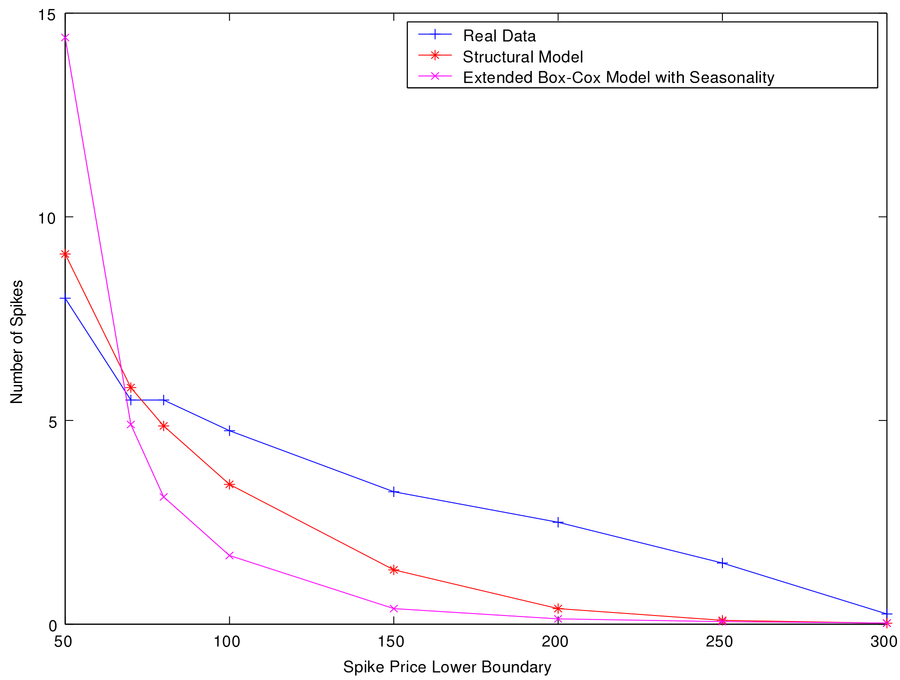

Figure 5. The Lower Boundaries of Spikes and the Number of Spikes

To compare the structural model and the extended Box-Cox transformation model with demand seasonality, we illustrate, in Figure 5, the number of spikes in summer per annum of the structural model, the extended Box-Cox transformation model with seasonality, and the historical data by changing the minimum size of the spikes from $\$50/MWh$ to $\$300/MWh$. As we have already discussed, at around $\$50/MWh$, the extended Box-Cox transformation model with demand seasonality exhibits twice as many spikes as the historical data and the structural model. This is due to the curvature of the Box-Cox transformation function. In contrast, above $\$70/MWh$, fewer price spikes are generated by the extended Box-Cox model than by the historical data and the structural model. This is because the slope of the supply curve generated by the extended Box-Cox model is much flatter than that generated by the structural model and the actual supply curve. While somewhat fewer spikes are generated in the structural model than in the historical data in the region between $\$100/MWh$ and $\$300/MWh$, the number of spikes generated by the structural model is closer to that of the historical data than that generated by the Box-Cox model, especially in the region between $\$50/MWh$ and $\$100/MWh$.

These results suggest that while demand seasonality is essential to describe the price spikes accurately, the structural model can generate price spikes much closer to the actual data than can the Box-Cox transformation model, even when the Box-Cox transformation model is extended to explicitly incorporate demand seasonality.

4. Application to Optimal Power Generation

4.1. Price Spikes and Optimal Operation

It is important for profitable power generation to take account of the possibility of price spikes. Thompson, Davison, and Rasmussen (2003) is an attempt to analyze optimal power generation in such a situation. Using the jump diffusion model of electricity prices, it considers a pump-storage hydropower plant, which controls the generating and pumping rates of various water flows, and derives an optimal water flow policy.

In this section, we derive an optimal operating policy for a pump-storage hydropower plant based on the structural model of electricity prices, and compare the result with that based on the jump diffusion model. The structural model relates electricity price with demand, which can be well described by a simple diffusion process. Thus, it can describe price spikes in terms of demand much more easily than the jump diffusion model. This simplicity of the structural model enables us to obtain an intuitive optimal policy to pump-up water when the demand level goes up, or when the probability of price spikes goes up. It also enables us to detect the trade-off between pumping-up water to speculate on the possibility of generating electricity at the high "spiked" prices and releasing water to sell electricity at the current price. This result turns out to be qualitatively very different from the optimal policy obtained by Thompson, Davison, and Rasmussen (2003) with the jump diffusion model.

4.2. The Jump Chance

To obtain the optimal operating policy, it is important to know how probable it is that price spikes will occur. We denote by $J_C(P_t, \acute{P}_T)$ the probability that the price P_T at T is higher than $\acute{P}_T$ conditional on the price P_t at t. That is,

$$J_C(P_t, \acute{P}_T) \equiv Prob(P_T \geq \acute{P}_T \mid P_t).$$

We call $J_C(P_t, \acute{P}_T)$ the Jump Chance at P_t. In the structural model, since electricity prices are linked to demand through the supply curve, the Jump Chance is given by the probability that demand D_T at time T exceeds $\acute{D}_T$ where $\acute{D}_T$ corresponds to the price $\acute{P}_T$. Note that we formulate $D_T = X_T + \bar{D}_T$, X follows an O-U process, and that

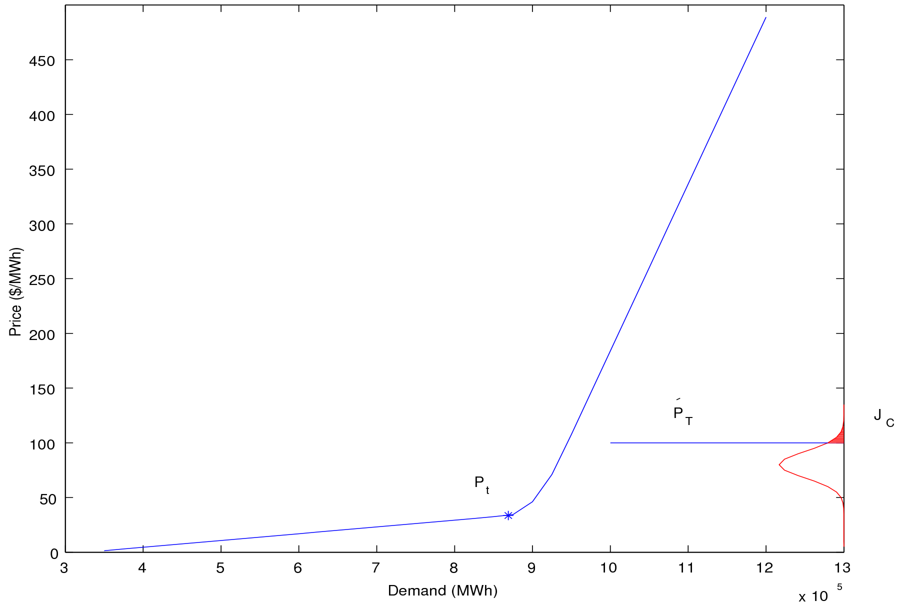

Figure 6. Jump Chance

$$X_T = e^{-\lambda_X(T-t)}X_t + \frac{\mu_X}{\lambda_X}[1 - e^{-\lambda_X(T-t)}] + \sigma_X \int_t^T e^{-\lambda_X(T-u)} dw_{1u}, \tag{19}$$

where we have $X_T \sim N(\mu_{X1}, \sigma_{X1}^2)$ with

$$\mu_{X1} = e^{-\lambda_X(T-t)}X_t + \frac{\mu_X}{\lambda_X}[1 - e^{-\lambda_X(T-t)}] \tag{20}$$

$$\sigma_{X1}^2 = \frac{\sigma_X^2}{2\lambda_X}[1 - e^{-2\lambda_X(T-t)}]. \tag{21}$$

Thus, in the structural model, the Jump Chance $J_C(P_t, \acute{P}_T)$ has the following simple expression

$$J_C(P_t, \acute{P}_T) = Prob(P_T \geq \acute{P}_T \mid P_t) = Prob(D_T \geq \acute{D}_T \mid D_t) = 1 - \Phi(\frac{\acute{X}_T - \mu_X}{\sigma_X}) \tag{22}$$

where Φ is the cumulative standard normal distribution function.

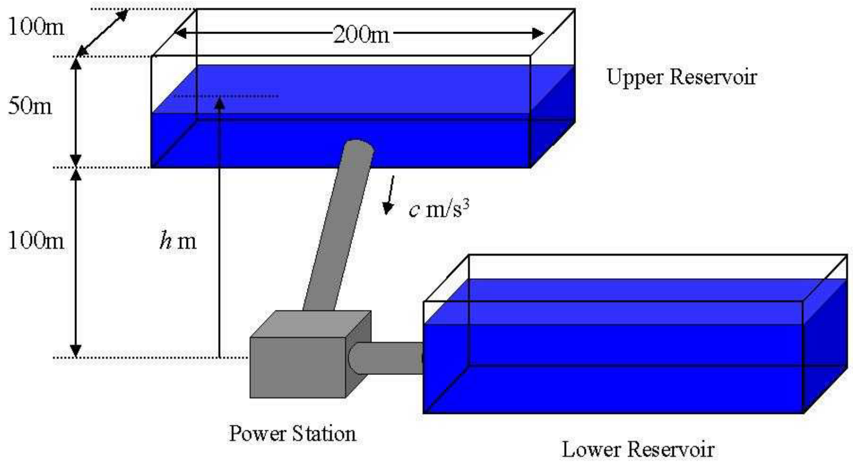

Figure 7. A Pump-Storage Facility

4.3. Optimal Operation of a Pump-Storage Hydropower Generator

Next, we set up the problem of the optimal operation of a pump-storage hydropower facility. To calculate the present value of future revenues, we need the electricity prices, the quantity of electricity generated by the facility, and the discount factor. We calculate the electricity price P_s at s from the structural model.

For the electricity generated by the facility, we consider the following hydropower facility considered by Thompson, Davison, and Rasmussen (2003) (See Figure 7). This facility can generate power by dropping water from a height while it also can pump water up using electricity. We denote by h_s the height of the water level in the upper reservoir at s, by c_s the flow rate of water for generation or pumping at s[5], and by $H(h_s, c_s)$ the amount of electricity generated by the facility at s. The revenue at s is thus given by $H(h_s, c_s)P_s$. Finally, we denote by Λ_s the stochastic discount factor at s. We choose Λ_s using the "Minimal Martingale Measure" of Schweizer (1991), which implies in our setting that the price of the risk not spanned by the stock market is zero [6].

The optimal operational problem of generating or pumping water flow rates $c_s \quad (t \leq s \leq T)$ to maximize the total present value of the revenues from time t to T is obtained by

$$\underline{C}_t = \max_{\{c_s, t \leq s \leq T\}} E_t[\int_t^T \frac{\Lambda_s}{\Lambda_t} H(h_s, c_s) P_s ds]. \tag{23}$$

We assume that the stock is traded and its price follows equation (24).

$$\frac{dS_t}{S_t} = \mu_s dt + \sigma_s dw_t \tag{24}$$

The PDE for the total present value of future revenues $\underline{C}$ is given by equation (25). The details are in Appendix B.

$$\begin{aligned} P_t H_t(c_{\max}) + \frac{dh(c_{\max})}{dt}\frac{\partial \underline{C}}{\partial h} - r\underline{C} + \frac{\partial \underline{C}}{\partial t} + \frac{1}{2}\sigma_s^2 S^2 \frac{\partial^2 \underline{C}}{\partial S^2} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 \underline{C}}{\partial X^2} + \rho\sigma_s\sigma_X S_t \frac{\partial^2 \underline{C}}{\partial X \partial S} \\ = -rS\frac{\partial \underline{C}}{\partial S} + (\phi\rho\sigma_X - \mu_X + \lambda_X X)\frac{\partial \underline{C}}{\partial X} \end{aligned} \tag{25}$$

The operation strategy $c_{\max}$ is given by the solution of the maximization problem (See also Appendix B).

$$\max_{\{c_t\}} (P_t H_t(c_t) + \frac{dh(c_t)}{dt}\frac{\partial \underline{C}}{\partial h}) \tag{26}$$

$$s.t. \quad c_{\min}(h,t) \leq c_t \leq c_{\max}(h,t) \tag{27}$$

where the terminal condition is given by

$$\underline{C}(S, X, h, T) = 0, \tag{28}$$

and $c_{\min}(h,t)$ and $c_{\max}(h,t)$ are exogenously given by the law of physics.

4.4. Simulation of the Optimal Operation

We simulate the optimal operation of the generating or pumping flow rates to maximize the total present value of the revenue. To calculate the electricity prices, we choose the period that corresponds to that beginning on July 1 and ending on July 30. For the power generation, we use the same model as Thompson, Davison, and Rasmussen (2003) (See Figure 7). According to them, the daily power $H_t(h,c)$ of the facility for generation and pumping is given by

$$H_t(c,h) = .0098hc\eta(h,c)\cdot 24, \quad 0 \le c \le \sqrt{2gh}, \quad 100 < h \le 150 \tag{29}$$

or

$$= -15\cdot 24, \quad c = \frac{-15\cdot .75}{.0098(2h-100)}, \quad 100 \le h < 150 \tag{30}$$

where the generation efficiency is assumed to be a quadratic function whose upper limit is 0.85.

$$\eta(h,c) = -0.85(\frac{.0098hc}{60} - 1)^2 + .85 \tag{31}$$

By the law of physics, we have

$$\frac{dh}{dt} = -4.32c. \tag{32}$$

The present value is obtained by the solution of the PDE in equation (25) with these conditions (from (29) to (32)) in Appendix B. The optimal control is given by

$$\max_{\{c_t\}}(P_tH_t(c_t) - 4.32c_t\frac{\partial \underline{C}}{\partial h}) \tag{33}$$

$$\text{s.t.} \quad c_t = \frac{-15(0.75)}{.0098(2h_t-100)} \quad \text{or} \quad 0 \le c_t \le \sqrt{2gh_t}. \tag{34}$$

As in Thompson, Davison, and Rasmussen (2003), the boundary condition is

$$\underline{C}(S,X,h,T) = 0. \tag{35}$$

Furthermore, the stock prices (S) and demand deviations (X) satisfy the following Neumann conditions. On the upper and lower bounds of the calculation area, the derivatives are zeros. On the water level (h), the facility can only generate the electricity ($c \geq 0$) if the water level is the highest $h = 150$. If the water level is the lowest, i.e., $h = 100$, the facility can only pump up water ($c < 0$).

We employ the closing prices of the S&P500 as stock prices for the PDE calculation. The stock price is assumed to follow equation (24). We estimate the parameters using a maximum likelihood method and obtain $\mu_S = 8.54 * 10^{-5}$ and $\sigma_S = 1.31 * 10^{-2}$. We assume that the risk free rate is 0.03 per annum, the correlation between stock prices and demand deviations is 0.5, and average daily demand ($\bar{D_t}$) is a constant 900,000MWh during this month. In making the valuation of the pump-storage facility, we select the SDF so that it is equal to the Minimal Martingale Measure.

The present values and the optimal operation policies at time 0 are shown in Figure 8 and Figure 9. Figure 8 describes the present value with two initial values: one is the height of the water level in the upper reservoir, the other is the demand for electricity ranging from 200,000MWh to 1,200,000MWh. Figure 9 shows the optimal operation policy at time 0 with the same initial value. The plus sign indicates power generation and the minus sign indicates pumping up water. Figure 8 shows that the present value increases as the water level rises. Figure 9 shows that the optimal strategy is to pump when both the water level and the demand for electricity are low. These results are consistent with Thompson, Davison, and Rasmussen (2003).

Figure 9 also reveals that the optimal strategy is to pump up water when the demand for electricity is between 850,000MWh and 950,000MWh, except when the water level of the upper reservoir is close to 150m, the upper limit. This result, however, shows the complexity of the optimal operation policy. Indeed, when the initial water level is relatively high, between $h = 120$ and 140, as the initial demand increases from the minimum, the optimal policy becomes to release water and generate electricity, but once the demand reaches the

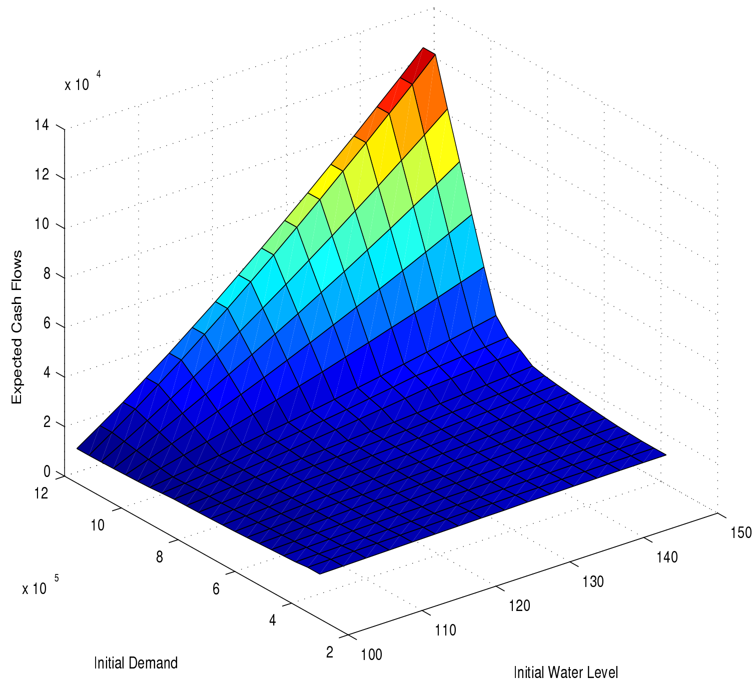

Figure 8. The Present Value of a Facility ($)

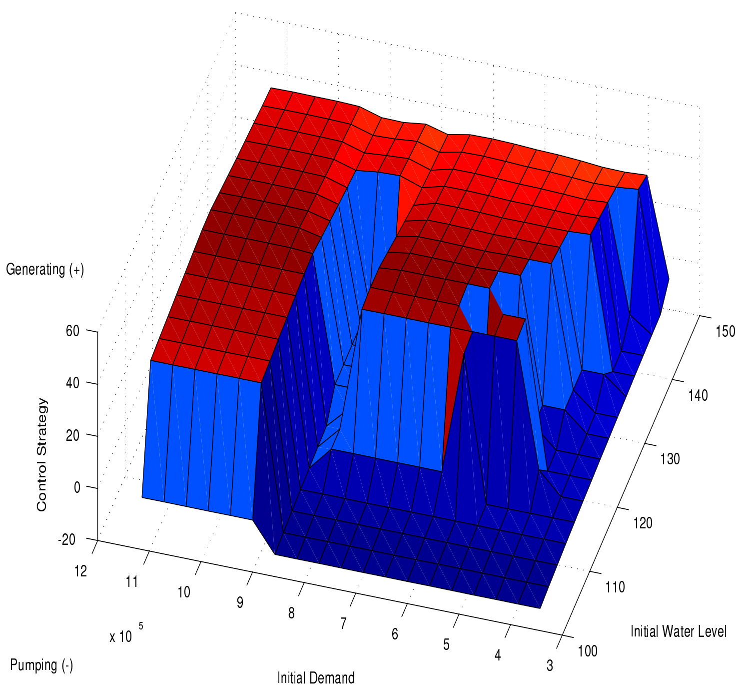

Figure 9. Optimal Operation (m^3/s)

level of 850,000MWh, the probability of price spike becomes so high that it is optimal to stop dropping water and to start pumping up. This trade-off is in sharp contrast with Thompson, Davison, and Rasmussen (2003) in which the optimal policy is not much affected by the level of demand.

The difference may owe partly to the fact that the parameter values in the structural model are estimated from the actual data while those in the jump diffusion model are arbitrarily set. However, we suspect that it is mainly due to the characteristics of the structural model and the jump diffusion model. Let us think of the case with the initial water level between 120m and 140m in the structural model. A price spike is highly probable in the demand range between 850,000MWh and 950,000MWh, where the slope of the supply curve suddenly increases. As the demand increases gradually from the minimum level, while dropping water down generates the revenue, the probability of price spikes (i.e., the potential profit of pumping up water and speculating on price spikes) increases gradually. Such gradual change of the trade-off between dropping water and pumping up water generates a complex optimal policy as depicted in Figure 9. On the other hand, in the regime-switching jump diffusion model of Thompson, Davison, and Rasmussen (2003), the probability of a price jump suddenly increases from almost 0 to close to 1, once price (or demand) exceeds a threshold value. Though, presumably, gradual increase in price (or demand) from the minimum level gradually increases the probability of price spikes, the sudden increase in the jump probability at the threshold value seems too dominant to generate the trade-off between dropping water and pumping up water. This leads to a simple optimal policy in Thompson, Davison, and Rasmussen (2003).

Recall that the jump diffusion model of Thompson, Davison, and Rasmussen (2003) requires a complicated combination of two jump processes in order to generate a realistic price spike. In contrast, the structural model needs only a simple diffusion setup as explained above. It is now clear that the simple structural model easily incorporates the relationship between the demand and the possibility of price spikes to describe a complex optimal strategy. The complicated jump diffusion model, however, seems to generate a simple optimal policy that directs the operation of the power facility to be almost independent of the price (or demand) level, which may be misleading for the actual use.

Finally, the structural model enables us to characterize easily the relationship between the Jump Chance - the possibility of price spikes - and the optimal strategy. We depict it in Figure 10, where the Jump Chance is taken to be the probability of exceeding $\$100/MWh$ tomorrow. Figure 10 shows that generation is optimal when the Jump Chance is less than 10% or more

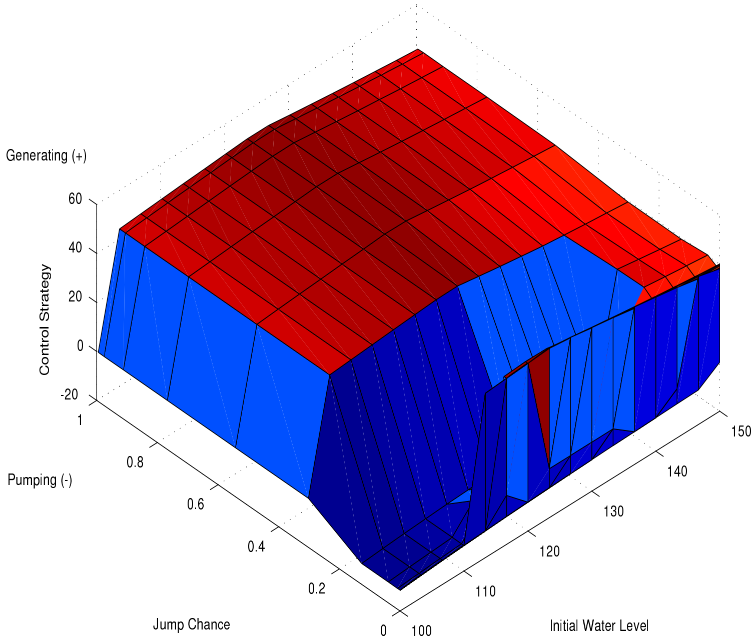

Figure 10. Jump Chance and Optimal Operation (m^3/s)

than 40% and pumping is optimal when the Jump Chance is between 10% and 40% [7].

5. Conclusion and Further Discussion

In this paper, we develop the structural model for price spikes in the electricity market. Using the data from the PJM markets, we compare the performance of the structural model with that of other models, such as the jump diffusion model and the Box-Cox transformation model.

The structural model can describe the occurrence of price spikes more accurately than the jump diffusion model. This is because the former directly captures the relationship between electricity demand and price spikes, while the latter does not.

The Box-Cox transformation model shares with the structural model the spirit of determining price by supply and demand. We show that the Box-Cox transformation model with demand seasonality describes price spikes more accurately than that without demand seasonality. That is, for accurate prediction of price spikes, it is important to treat explicitly the seasonality of demand in the model. However, even though it explicitly includes seasonality, the Box-Cox transformation model cannot describe price spikes as accurately as the structural model at least for the PJM market. This is because the curvature of the assumed functional form of the supply curve in the Box-Cox transformation is too small to capture the sudden change in slope of the actual supply curve at the threshold of the supply. The change is due to the constitution of generating facilities. The structural model can describe this change in slope much better than the Box-Cox model.

We solve the optimal operation problem of a hypothetical pump-storage hydropower generator. The optimal strategy includes to pump water up to the upper reservoir in order to prepare for a possible profit opportunity when the probability of the price spikes is high. This complicates the optimal operation policy. Indeed, the result obtained by the structural model estimated from the actual data reveals the complex trade-off between dropping water and pumping up water in the optimal policy. This is in sharp contrast with the simple optimal policy shown by Thompson, Davison, and Rasmussen (2003) based on the jump diffusion model with the hypothetical parameters. While this result can be taken as a caveat against using the jump diffusion model without fitting the actual data, it also shows that the simple structural model can easily incorporate the relationship between the demand and the probability of the price spikes to describe the complex optimal policy, whereas the complex jump diffusion model can generate only the simple optimal policy, which may be misleading for the actual use.

Finally, several issues are left for future investigation. First, although we assume that the supply curve is fixed, the curve may actually change due to fluctuations in the cost of energy, availability of resources, and changes in the facility constitution. Indeed, in the Nord Pool where hydropower generators are dominant, the supply curve fluctuates depending on the amount of rainfall. For more accurate risk management, it may be desirable to incorporate the shift of supply curve into the model. Moreover, in the model above, we ignore the weekend effect of electricity demand, which is found in the historical data. The incorporation of such effect may improve the accuracy of the model.

Appendix A. Parameter Estimation

Parameter Estimation of a Jump Diffusion Model

We show the method of estimating the parameters of a jump diffusion model and the results of the estimation. The transition density of the jump diffusion model is given by equation (A1).

$$
\begin{aligned}
f[P_t \mid P_{t-1}] = q \cdot \exp\left(\frac{-(\log(P_t) - \log(P_{t-1}) - \alpha(\beta - \log(P_{t-1}))\Delta t - \mu_J)^2}{2(v^2\Delta t + \sigma_J^2)}\right) \frac{1}{\sqrt{2\pi(v^2\Delta t + \sigma_J^2)}} \\
+ (1-q) \cdot \exp\left(\frac{-(\log(P_t) - \log(P_{t-1}) - \alpha(\beta - \log(P_{t-1}))\Delta t)^2}{2v^2\Delta t}\right) \frac{1}{\sqrt{2\pi v^2 \Delta t}} \qquad \text{(A1)}
\end{aligned}
$$

The maximum likelihood method is implemented by solving the maximization problem of equation (A2) with equation (A1).

$$\max_{\Theta} \sum_{t=1}^{T} \log(f[P_t \mid P_{t-1}]) \qquad \text{(A2)}$$

where $\Theta = \{\alpha, \beta, v^2, q, \mu_J, \sigma_J^2\}$. The results of the estimates are presented in Table 5. According to

Parameter	α	β	v^2	q	μ_J	σ_J^2
Estimate	139.69	3.06	25.99	0.06	0.55	0.64
t-statistics	9.49	102.55	12.25	2.74	2.49	10.64
Log-likelihood	-183.51					
SIC	367.02					
AIC	379.02					

Table 5
Parameter Estimation by MLE (Jump Diffusion Model)

t-statistics, all parameters are significant. The probability of jumps is 6% per day.

Parameter Estimation of a Box-Cox Transformation Model

We show the method of estimating the parameters of a Box-Cox transformation model and the results of the estimation. The transition density $q(x,y)$ of Y is given by

$$q(x,y) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{(y-x\rho-b)^2}{2\theta}}. \tag{A3}$$

With Jacobian, the transition density $p(x,y)$ of P leads to

$$p(x,y) = |g'_\alpha(y)| \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{(g_\alpha(y)-g_\alpha(x)\rho-b)^2}{2\theta}}. \tag{A4}$$

Therefore the log-likelihood is

$$\tilde{L} = \sum_{i=1}^{n} \log|g'_\alpha(P_i)| - \frac{n}{2}\log(2\pi\theta) - \frac{1}{2\theta}\sum_{i=1}^{n}(g_\alpha(P_i) - g_\alpha(P_{i-1})\rho - b)^2. \tag{A5}$$

By maximizing the log-likelihood $\tilde{L}$, we have the parameters of α, b, ρ, and θ. The results are presented in Table 6. According to t-statistics, all parameters are significant.

Parameter	α	b	ρ	θ
Estimate	-0.66	0.44	0.66	$1.47 * 10^{-3}$
t-statistics	-16.57	11.43	27.71	3.80
Log-likelihood	-2460.81			
SIC	4947.98			
AIC	4929.61			

Table 6
Parameter Estimation by MLE (Box-Cox Transformation Model)

Parameter Estimation of an Extended Box-Cox Transformation Model with Seasonality

We use a nonlinear least square method to estimate the parameters. The result is presented in Table 7. Judging from the *t*-statistics, all parameters are statistically significant.

Parameter	α	b	c
Estimate	-0.53	$-1.23*10^6$	$1.28*10^6$
t-statistic	-10.17	-3.56	4.28
Log-likelihood	-3295.89		
SIC	6611.56		
AIC	6597.77		

Table 7
Estimation of an Extended Box-Cox Transformation Model with Seasonality

Appendix B. Optimal Operation of a Pump-Storage Facility

Derivation of PDE

We assume that the stochastic discount factor (SDF:Λ_s) is selected as the Mimimal Martingale Measure, which means that only the marketed risk is priced.

$$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \phi dw_t \tag{B6}$$

We consider the optimal operation problem for a pump-storage hydropower plant whose total present value is maximized by selecting the generating or pumping water flow $c_s \quad (t \leq s \leq T)$.

$$\underline{C}_t = \max_{\{c_s, t\leq s\leq T\}} E_t\left[\int_t^T \frac{\Lambda_s}{\Lambda_t} H(h_s, c_s) P_s ds\right] \tag{B7}$$

Then, we have

$$0 = \max_{\{c_t\}}\left(\frac{H(h_t, c_t)P_t}{\underline{C}_t} dt + \frac{E_t[d(\Lambda \underline{C}_t)]}{\Lambda \underline{C}_t}\right). \tag{B8}$$

By Ito's Lemma, we have

$$0 = \max_{\{c_t\}}\left(E_t\left[\frac{d\underline{C}_t}{\underline{C}_t}\right] + \frac{H(h_t, c_t)P_t}{\underline{C}_t} dt - rdt + E_t\left[\frac{d\Lambda}{\Lambda}\frac{d\underline{C}_t}{\underline{C}_t}\right]\right). \tag{B9}$$

It is assumed that $\underline{C}$ follows

$$\frac{d\underline{C}}{\underline{C}} = \mu_{\underline{C}} dt + \sigma_{\underline{C}w} dw_t + \sigma_{\underline{C}z} dz_t. \tag{B10}$$

Then, we have

$$0 = \max_{\{c_t\}}\left(\mu_{\underline{C}} + \frac{H(h_t, c_t)P_t}{\underline{C}_t} - r - \phi\sigma_{\underline{C}w}\right). \tag{B11}$$

Ito's Lemma gives the PDE of $\underline{C}$.

$$\begin{aligned} P_t H_t(c_{\max}) + \frac{dh(c_{\max})}{dt}\frac{\partial \underline{C}}{\partial h} - r\underline{C} + \frac{\partial \underline{C}}{\partial t} + \frac{1}{2}\sigma_s^2 S^2 \frac{\partial^2 \underline{C}}{\partial S^2} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 \underline{C}}{\partial X^2} + \rho\sigma_s\sigma_X S_t \frac{\partial^2 \underline{C}}{\partial X \partial S} \\ = -rS\frac{\partial \underline{C}}{\partial S} + (\phi\rho\sigma_X - \mu_X + \lambda_X X)\frac{\partial \underline{C}}{\partial X} \end{aligned} \tag{B12}$$

Thus, the equation (25) is derived.

Physical Conditions of Generating and Pumping and Water Level Change

We show the physical conditions of the flow rates (c) of generating and pumping and water level change (dh). The conditions follow Thompson, Davison, and Rasmussen (2003).

We present physical conditions of c required in equation (26).The flow velocities on the surface of the upper reservoir and the turbine center of the generator are denoted by v_1 and v_2, the heights by h_1 and h_2, and the pressures by P_1 and P_2. ρ is the density of the water. Bernoulli's law is applied. We have $\frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2 = \frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1$. We set the rate of flow as $c(m^3/s)$. The conservation of mass leads to $c = a_1 v_1 = a_2 v_2$. We assume that the surface area of the upper reservoir is $a_1 = 20,000$ (width $100m$ and length $200m$), that of the generator at the turbine center is $a_2 = 1$, and the difference between the water levels is $h = h_1 - h_2$. The condition is given by $c_{\max} \approx \sqrt{2gh}$. In contrast, the power required to pump is assumed to be 15 MWh and its efficiency is assumed to be 75%. In order to pump water up, potential energy of $2h - 100$ is needed with the physical condition$.75 = \frac{.0098(2h-100)c}{-15}$. With these assumptions and conditions, we describe the daily power in the following way

$$
\begin{aligned}
H_t(c,h) &= .0098hc\eta(h,c)\cdot 24, \quad 0 \le c \le \sqrt{2gh}, \quad 100 < h \le 150 \\
&\text{or} \\
&= -15\cdot 24, \quad c = \frac{-15\cdot .75}{.0098(2h-100)}, \quad 100 \le h < 150.
\end{aligned} \tag{B13}
$$

We assume that the inflow to the upper reservoir is 0.

Finally, we formulate the change dh in the water level required to calculate equation (26). We indicate the relationship between the water level h and the flow rate of water c

$$dh \cdot a_1 = -c \cdot dt. \tag{B14}$$

Converting it to the daily amount, we get

$$\frac{dh}{dt} = \frac{-60\cdot 60\cdot 24}{200\cdot 100}c = -4.32c. \tag{B15}$$

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Notes

[1]Johnson and Barz (1999) compares four models: the normal process, the mean-reverting model, the lognormal process, and the lognormal mean-reverting process with jump processes. Deng (2000) compares three models: the constant volatility, the stochastic volatility, and the regime switching volatility with jump processes.

[2]PJM is a U.S. electricity market opened in 1997, which originally covered Pennsylvania, New Jersey, Maryland, Delaware, Virginia, and Washington. From April 2002, it also covers West Virginia.

[3]We identify four seasons. Spring is from March to May, summer is from June to August, autumn is from September to November, and winter is from December to February.

[4]We use round off in the first decimal place in consistency with the historical data.

[5]$c_s \geq$ means generating electricity. $c_s < 0$ means pumping water up using electricity.

[6]Cochrane and Saa-Requejo (2000) gives one method, called the Good-Deal Bounds, to price the risks that are not spanned by existing securities. Setting the exogenous Sharpe ratio equal to the security market price of risk ϕ, the stochastic discount factor from the Good-Deal Bounds is equivalent to that from the Minimal Martingale Measure.

[7]At first glance, one may wonder why the optimal strategy does not imply pumping up water at a very high Jump Chance, say 70% or 80%. To understand the reason, note that we define the Jump Chance as the probability of the price tomorrow being more than \$100$/MWh$. Due to this definition, today's price, whose Jump Chance is more than 40%, is already very high. Since it is very costly to pump up water by consuming electricity at such a high price, it is optimal for the company not to pump up water. This is why the optimal strategy is not to pump up water when the Jump Chance is more than 40%.