Exercise 1

1) Using Eviews (file named return.wf1), plot the ACF and PACF function for the series "returns". Identify the series.

2) Read the paper "Do we really know that financial markets are efficient?" by Lawrence H. Summers.

3)

i) Derive the theoretical autocorrelation function of an ARMA(1,2) process.

ii) Derive the theoretical autocorrelation function of an ARMA(2,1) process.

4) Derive the partial autocorrelation function of an AR(3) process.

5) The simplest Present Value Theory of stock prices says that the stock price is the discounted value of all future dividends. Assume that dividends follow an AR(2) process and that the discounted factor is constant.

i) Which process does stock prices follow?

Solution

1) In order to identify the series we are going to use the plot of the ACF and the PACF functions. In order to do that we have to use the "Correlogram" command.

You must choose the E-Views command "Quick", then "Series Statistics" and then "Correlogram". When you get there, you should specify the series you would like to plot and then the lags you would like to include (default=36) in the estimation.

When you plot the Correlogram for the series "Return" you will find the following results:

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|* |</td><td>.|* |</td><td>1</td><td>0.147</td><td>0.147</td><td>20.081</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>-0.014</td><td>-0.036</td><td>20.252</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>3</td><td>-0.043</td><td>-0.037</td><td>21.997</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>4</td><td>0.032</td><td>0.044</td><td>22.942</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>5</td><td>0.031</td><td>0.018</td><td>23.828</td><td>0.000</td></tr>
<tr><td>|. |</td><td>*|. |</td><td>6</td><td>-0.083</td><td>-0.093</td><td>30.280</td><td>0.000</td></tr>
<tr><td>|. |</td><td>.|. |</td><td>7</td><td>-0.081</td><td>-0.052</td><td>36.394</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>8</td><td>-0.039</td><td>-0.022</td><td>37.823</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>9</td><td>0.051</td><td>0.050</td><td>40.282</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>10</td><td>0.003</td><td>-0.014</td><td>40.293</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|* |</td><td>11</td><td>0.056</td><td>0.068</td><td>43.247</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>12</td><td>0.047</td><td>0.033</td><td>45.308</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|. |</td><td>13</td><td>0.066</td><td>0.045</td><td>49.433</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>14</td><td>0.041</td><td>0.019</td><td>50.992</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>15</td><td>0.009</td><td>0.007</td><td>51.065</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>16</td><td>-0.010</td><td>-0.009</td><td>51.154</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>17</td><td>-0.046</td><td>-0.037</td><td>53.137</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>18</td><td>-0.012</td><td>0.004</td><td>53.285</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>19</td><td>0.023</td><td>0.040</td><td>53.799</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>20</td><td>0.017</td><td>0.012</td><td>54.069</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>21</td><td>0.015</td><td>0.021</td><td>54.294</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>22</td><td>-0.047</td><td>-0.055</td><td>56.366</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>23</td><td>-0.006</td><td>-0.005</td><td>56.403</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>24</td><td>-0.015</td><td>-0.033</td><td>56.624</td><td>0.000</td></tr>
</table>

To identify a series we know that:

- An AR(p) process has a declining AC function and the PACF is zero for lags greater than p.
- A MA(q) process has a AC function that is zero for lags greater than q and a PACF that declines exponentially.

The autocorrelation is described by:

$$\rho(k) = \frac{Cov(\times_t, \times_{t-k})}{(V(x_t)V(x_{t-k}))^{1/2}} = \frac{\gamma(k)}{\gamma(0)}$$

Looking at the table, we can see that all the lags are significant and therefore you reject the null hypothesis of $\rho(k) = 0$ for each lag, and as a results the residual cannot be a white noise process. Notice that the estimated value for Q-Stat $\rho(1)$ it is significant at a 95%, which implies that the null hypothesis of $\rho(1) = 0$.is rejected.

We will attempt to identify the series from specific to general. We will fit an AR(1) and check whether the residuals of that regression are white noise.

Since the estimate of $\rho(1)$ is too small it is very difficult (looking at the above table) to tell which type of process is the best to characterize the series under scrutiny (a MA or AR process). We first try with an AR(1).

$$y_t = \phi_1 y_{t-1} + \varepsilon_t$$

You should go to "Quick", "Estimate Equation", an then you have to type the dependent and independent variables that you will include in the regression: *Returns C AR(1).* We obtain the following results:

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000542	0.000272	1.995215	0.0463
AR(1)	0.147150	0.032482	4.530161	0.0000
R-squared	0.021659	Mean dependent var		0.000543
Adjusted R-squared	0.020604	S.D. dependent var		0.007132
S.E. of regression	0.007058	Akaike info criterion		-7.067120
Sum squared resid	0.046181	Schwarz criterion		-7.056713
Log likelihood	3284.677	F-statistic		20.52236
Durbin-Watson stat	1.989755	Prob(F-statistic)		0.000007
Inverted AR Roots	.15			

The series "returns" seem to be an AR(1) which seems to be stationary because the AR root is $\frac{1}{0.15} > 1$. Nevertheless we can only be sure that it is an AR(1) if the residuals are clean, therefore we need to check the correlogram of the residual to inquire whether the residuals are (or not) a white noise process. If they are, we stop and conclude the process is an AR(1), if not we continue augmenting the model.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|. |	.|. |	1	0.005	0.005	0.0223	
.|. |	.|. |	2	-0.030	-0.030	0.8505	0.356
.|. |	.|. |	3	-0.048	-0.048	2.9962	0.224
.|. |	.|. |	4	0.035	0.035	4.1439	0.246
.|. |	.|. |	5	0.040	0.037	5.6272	0.229
|. |	*|. |	6	-0.079	-0.080	11.503	0.042
|. |	*|. |	7	-0.066	-0.061	15.612	0.016
.|. |	.|. |	8	-0.036	-0.038	16.852	0.018
.|. |	.|. |	9	0.059	0.046	20.100	0.010
.|. |	.|. |	10	-0.012	-0.017	20.246	0.016
.|. |	.|. |	11	0.051	0.061	22.686	0.012
.|. |	.|. |	12	0.030	0.036	23.560	0.015
.|. |	.|. |	13	0.056	0.048	26.501	0.009
.|. |	.|. |	14	0.031	0.026	27.418	0.011
.|. |	.|. |	15	0.004	0.012	27.437	0.017
.|. |	.|. |	16	-0.005	-0.002	27.456	0.025
.|. |	.|. |	17	-0.044	-0.037	29.317	0.022
.|. |	.|. |	18	-0.010	-0.007	29.409	0.031
.|. |	.|. |	19	0.024	0.037	29.940	0.038
.|. |	.|. |	20	0.012	0.015	30.072	0.051
.|. |	.|. |	21	0.020	0.032	30.471	0.063
.|. |	.|. |	22	-0.050	-0.049	32.871	0.048
.|. |	.|. |	23	0.003	-0.007	32.879	0.064
.|. |	.|. |	24	-0.007	-0.026	32.932	0.082

We can see that for the 6th lag, the Prob(associated with the Q statistics) are smaller than the 5% critical level and therefore the Q-Stat determines that the lag 6 is significant. This test shoews that in the sixth lag there is either an AR or a MA component. We estimate the following equation:

$$y(t) = \phi_1 y_{t-1} + \phi_6 y_{t-6} + \varepsilon_t$$

To estimate this model we go in the Eviews menu to "Quick", "Estimate Equation" and type: *Return C AR(1) AR(6)*, obtaining the following results:

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000541	0.000246	2.200320	0.0280
AR(1)	0.149802	0.032386	4.625564	0.0000
AR(6)	-0.087959	0.032345	-2.719380	0.0067
R-squared	0.029410	Mean dependent var		0.000543
Adjusted R-squared	0.027314	S.D. dependent var		0.007132
S.E. of regression	0.007034	Akaike info criterion		-7.072921
Sum squared resid	0.045815	Schwarz criterion		-7.057311
Log likelihood	3288.372	F-statistic		14.02948
Durbin-Watson stat	2.000263	Prob(F-statistic)		0.000001
Inverted AR Roots	.60 -.33i	.60+.33i	.02+.66i	.02 -.66i
	-.55 -.33i	-.55+.33i		

The t-statistics for theAR(1) and AR(6) lags are both significant at a 5% (NB these are only valid when the residuals are white noise, otherwise you should use Newey and West standard errors). Looking at the Correlogram of the residuals of this regresion realise that we cannot reject the null hypothesis taht the residual is a white noise process. We therefore stop the identification procedure and conclude that the above model characterize the data correctly.

The Correlogram shows that we cannot reject the null hypothesis that states that the residuals are a white noise process.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|. |	.|. |	1	0.000	0.000	0.0001	
.|. |	.|. |	2	-0.029	-0.029	0.7955	
.|. |	.|. |	3	-0.047	-0.047	2.8766	0.090
.|. |	.|. |	4	0.032	0.031	3.8171	0.148
.|. |	.|. |	5	0.045	0.043	5.7335	0.125
.|. |	.|. |	6	0.012	0.012	5.8642	0.210
.|. |	.|. |	7	-0.048	-0.043	8.0545	0.153
.|. |	.|. |	8	-0.035	-0.031	9.1906	0.163
.|. |	.|. |	9	0.056	0.052	12.120	0.097
.|. |	.|. |	10	-0.012	-0.020	12.245	0.141
.|. |	.|. |	11	0.052	0.054	14.746	0.098
.|. |	.|. |	12	0.024	0.034	15.279	0.122
.|. |	.|. |	13	0.052	0.055	17.840	0.085
.|. |	.|. |	14	0.029	0.031	18.630	0.098
.|. |	.|. |	15	0.010	0.010	18.724	0.132
.|. |	.|. |	16	-0.009	-0.005	18.807	0.172
.|. |	.|. |	17	-0.040	-0.043	20.342	0.159
.|. |	.|. |	18	-0.008	-0.014	20.403	0.203
.|. |	.|. |	19	0.025	0.026	20.996	0.226
.|. |	.|. |	20	0.009	0.005	21.077	0.276
.|. |	.|. |	21	0.018	0.027	21.377	0.316
.|. |	.|. |	22	-0.051	-0.050	23.838	0.250
.|. |	.|. |	23	0.001	-0.002	23.838	0.301
.|. |	.|. |	24	-0.006	-0.021	23.875	0.354

Nevertheless, since we were not sure about the nature of the sixth lag (it seems that it could either be an AR(6) or a MA(6)) we ill also estimate the following model :

$$y(t) = \phi_1 y_{t-1} + \theta_6 \varepsilon_{t-6} + \varepsilon_t$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000539	0.000250	2.155296	0.0314
AR(1)	0.145798	0.032547	4.479613	0.0000
MA(6)	-0.075680	0.032824	-2.305661	0.0213
R-squared	0.027516	Mean dependent var		0.000543
Adjusted R-squared	0.025416	S.D. dependent var		0.007132
S.E. of regression	0.007041	Akaike info criterion		-7.070972
Sum squared resid	0.045904	Schwarz criterion		-7.055361
Log likelihood	3287.467	F-statistic		13.10050
Durbin-Watson stat	1.989908	Prob(F-statistic)		0.000002
Inverted AR Roots	.15			
Inverted MA Roots	.65	.33+.56i	.33 -.56i	-.33 -.56i
	-.33+.56i	-.65		

The residuals of the above regression seem to be a white noise process and therefore we can conclude that this model also seems to fit the data correctly/

Selection Criteria

Since both models seem to fit the series, we choose the model that has the lower value for the relevant "Selection Criteria". We consider the Akaike info criterion (AIC) and Schwarz criterion (SCH) (NB: To be sure whether you have to choose the minimum or the maximum, always check the way the program that you are using computes this criteria).

The above selection criteria seem to the AR(1), AR(6) model for the series "returns".

Forecasting Criteria

Another way of comparing models is to check which model produces the best forecast out of sample . To do this, once you have specified the AR(1) AR(6) model, go to Procs/Forecast, type a name for the forecasted serie (default: returnsf). and choose the dynamic forecasting-method.and set the forecast period: 7920-7928.

You obtain the following results:

We seek to find models with small RMSE, MAE and MAPE values (the smaller these values the better forecasting ability).

The Theil Inequality Coefficient lies between $[0, 1]$ where 0 is represents a perfect fit.

The bias, variance and covariance proportion result from the decomposition of the Mean squared forecast error: $MSFE = \sum(\widetilde{y_t} - y_t)^2$

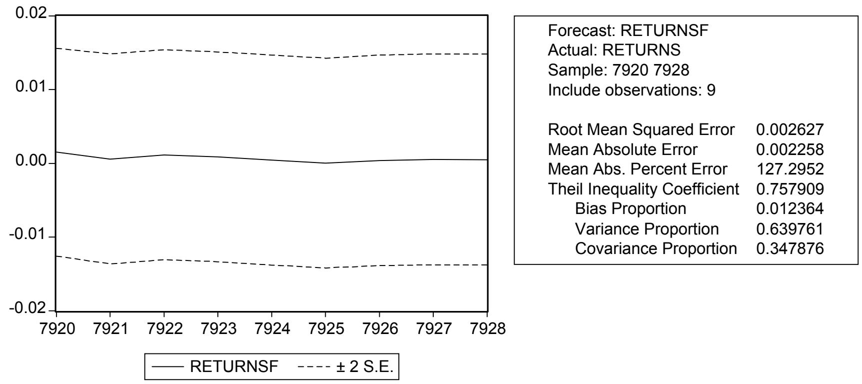

Figure 1:

The bias proportion tells us how far the mean of the forecast is from the mean of the series. The variance proportion tells us how far is the variance of the forecast from the actual and the covariance proportion measures the remaining unsystematic forecasting error.

The idea is to obtain the smaller values for the mentioned measures.

Now, we estimate the other model: AR(1) MA(6) and repeat the same procedure, we obtain

Comparing the values for the RMSE, MAE and MAPE we conclude that the model AR(1) MA(6) seems to perform better out of sample than the other model, but if you look at the values for the Theil Inequality, Bias and Variance Proportion, it seems that the model AR(1) AR(6) has a better out of sample performance.

3 ii) Consider the following ARMA(2,1)

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

To calculate the autocorrelation function we first compute the autocovariance function and then divide by the variance.

$$\gamma(k) = E(y_t y_{t-k}) = \phi_1 E(y_{t-1} y_{t-k}) + \phi_2 E(y_{t-2} y_{t-k}) + \theta_1 E(\varepsilon_{t-1} y_{t-k}) + E(\varepsilon_t y_{t-k})$$

To calculate the autocovariance function we need to give values to.$k = 0, 1, 2, ...$until we find a recursion which is valid for all $k >$ than an integral.

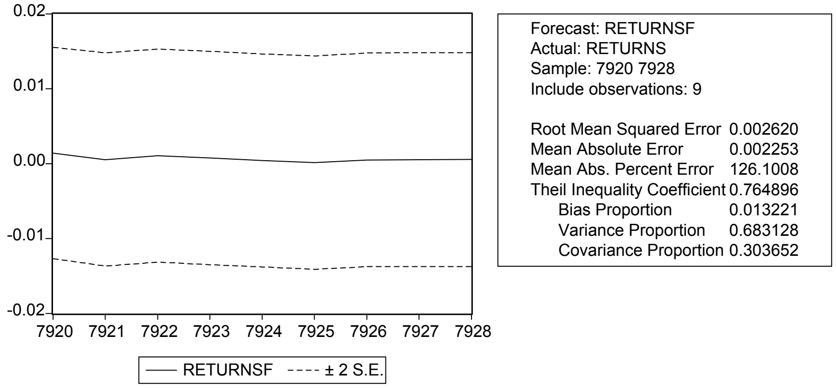

Figure 2:

$$\gamma(0) = E(y_t y_t) = \phi_1 E(y_{t-1} y_t) + \phi_2 E(y_{t-2} y_t) + \theta_1 E(\varepsilon_{t-1} y_t) + E(\varepsilon_t y_t) \qquad for\ k = 0.$$

which can be written as

$$\begin{aligned}\gamma(0) = E(y_t y_t) = \phi_1 \gamma(1) + \phi_2 \gamma(2) + \theta_1 E(\varepsilon_{t-1}(\phi_1 y_{t-1} + \phi_2 y_{t-2} + \theta_1 \varepsilon_{t-1} + \varepsilon_t)) \\ +E(\varepsilon_t(\phi_1 y_{t-1} + \phi_2 y_{t-2} + \theta_1 \varepsilon_{t-1} + \varepsilon_t)) \qquad for\ k = 0.\end{aligned}$$

which simplifies to

$$\gamma(0) = \phi_1 \gamma(1) + \phi_2 \gamma(2) + \theta_1(\phi_1 + \theta_1)\sigma_\varepsilon^2 + \sigma_\varepsilon^2 \qquad for\ k = 0.$$

analogously we can find that

$$\gamma(1) = \phi_1 \gamma(0) + \phi_2 \gamma(1) + \theta_1 \sigma_\varepsilon^2 \qquad for\ k = 1.$$

$$\gamma(2) = \phi_1 \gamma(1) + \phi_2 \gamma(0) \qquad for\ k = 2.$$

Using the last three equations we can obtain $\gamma(2)$, $\gamma(1)$ and $\gamma(0)$ as a function of $\phi_2,\ \phi_1,\ \theta_1\ and\ \sigma_\varepsilon^2$.

Notice also that $\gamma(k) = \phi_1 \gamma(k-1) + \phi_2 \gamma(k-2) \qquad for\ k > 2$. The autocorrelation function can be obtained simply by dividing by $\gamma(0)$.

4) The partial autocorrelation function is used to determine the number of autoregressive terms an ARMA model has. The usual practice is to derive the PACF of a high order AR and then determine empirically which lags are significant. These are sometimes called the Yule-Walker equations.

Consider an $AR(p)$

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \varepsilon_t$$

Then the Yule -Walker equations are

$$\rho(k) = \phi_1 \rho(k-1) + \phi_2 \rho(k-2) + ... + \phi_p \rho(k-p) \qquad for\ k \geq 1.$$

Giving values to $k = 1, ..., p.$ we can get a system of equations in $\phi_i(\rho(1), ...\rho(k))$ $for\ i = 1, ..., .p.$

$$\rho(1) = \phi_1 \rho(0) + \phi_2 \rho(1) + ... + \phi_p \rho(1-p) \qquad for\ k = 1.$$

$$\rho(2) = \phi_1 \rho(1) + \phi_2 \rho(0) + ... + \phi_p \rho(2-p) \qquad for\ k = 2.$$

....

....

$$\rho(p) = \phi_1 \rho(p-1) + \phi_2 \rho(p-2) + ... + \phi_p \rho(0) \qquad for\ k = p.$$

This system of equations have the partial autocorrelations as a solution $\phi_i(\rho(1), ...\rho(k))$ $for\ i = 1, ..., .p.$

Therefore an $AR(P)$ has p partial autocorrelations different from 0.

(5) The present value model states that the stock prices can be written as the discounted (at a constant rate) value of future dividends, i.e.

$$P_t = E_t \sum_{i=0}^{\infty} (\frac{1}{1+r})^i D_{t+i}.$$

It is also assumed that dividends follow an AR(2), i.e.,

$$D_t = \phi_1 D_{t-1} + \phi_2 D_{t-2} + \varepsilon_t$$

This expression can be written in the companion form as

$$\begin{bmatrix} D_t \\ D_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} D_{t-1} \\ D_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} \text{ or } Z_t = AZ_{t-1} + \zeta_t,$$

where $Z_t = \begin{bmatrix} D_t \\ D_{t-1} \end{bmatrix}$, $A = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$ and $\zeta_t = \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix}$.

Then to obtain an i periods ahead forecast of dividends it is straight forward since it can be written as

$$E_t D_{t+i}. = [1,0]\, E_t Z_{t+i} = [1,0]\, A^i Z_t$$

Then substituting in the present value expression we get

$$P_t = [1,0] \sum_{i=0}^{\infty} (\frac{1}{1+r})^i A^i Z_t.$$

Notice that $\sum_{i=0}^{\infty} (\frac{1}{1+r})^i A^i Z_t. = (I - (\frac{A}{1+r}))^{-1} Z_t$.

If we define a matrix $k = (I - (\frac{A}{1+r}))^{-1}$

Then $[1,0]\, (I - (\frac{A}{1+r}))^{-1} Z_t$ can be written as $k_{11} D_t + k_{12} D_{t-1}$, where k_{11} and k_{12} are the elements of the first row of k.

Then we can write the price equation as $P_t = k_{11} D_t + k_{12} D_{t-1,}$ or

$$P_t = (k_{11} + k_{12} L) D_t.$$

Since the Dividends process can be written as

$$D_t = \frac{\varepsilon_t}{1 - \phi_1 L - \phi_2 L^2},$$

then the price equation can be written as

$$P_t = (k_{11} + k_{12} L) \frac{\varepsilon_t}{1 - \phi_1 L - \phi_2 L^2}.$$

or

$$(1 - \phi_1 L - \phi_2 L^2) P_t = (k_{11} + k_{12} L) \varepsilon_t,$$

an $ARMA(2,1)$ process.