Diversification

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Overview

A. Diversification
B. Asset Allocation with 2 Risky Assets
C. Asset Allocation with 2 Risky and a Riskless Asset
D. Asset Allocation with Many Risky and a Riskless Asset

A. Diversification

Diversification

- Is there a way that investors can reduce the risk they face when investing in risky assets?
- *Diversification*: Spreading a portfolio over many different investments to avoid excessive exposure to any one source of risk.

"Do not put all your eggs in one Basket"

B. Asset Allocation with 2 Risky Assets

Diversification with 2 Assets: Exercise

- Suppose we have two assets, US and JP, with:

	mean	volatility
US	13.6%	15.4%
JP	15.0%	23.0%

and with correlation 27%.

- If an investor holds 60% in the US and 40% in JP what is the mean and volatility of the portfolio?
- 'volatility' is another word for 'standard deviation'

Diversification with 2 assets: Exercise

- Portfolio mean:

  $E(R_p) = 0.6*0.136 + 0.4*0.150 = 14.2\%$
- Portfolio variance:

  $var(R_p) = (0.6)^2*(0.154)^2 + (0.4)^2*(0.230)^2$

  $+2*0.6*0.4*0.27*0.154*0.230$

  $= 0.022$

  $\sigma_p = 14.7\%$
- This portfolio has higher expected return and lower risk than the US market alone!

Risk and Return with Varying Weights

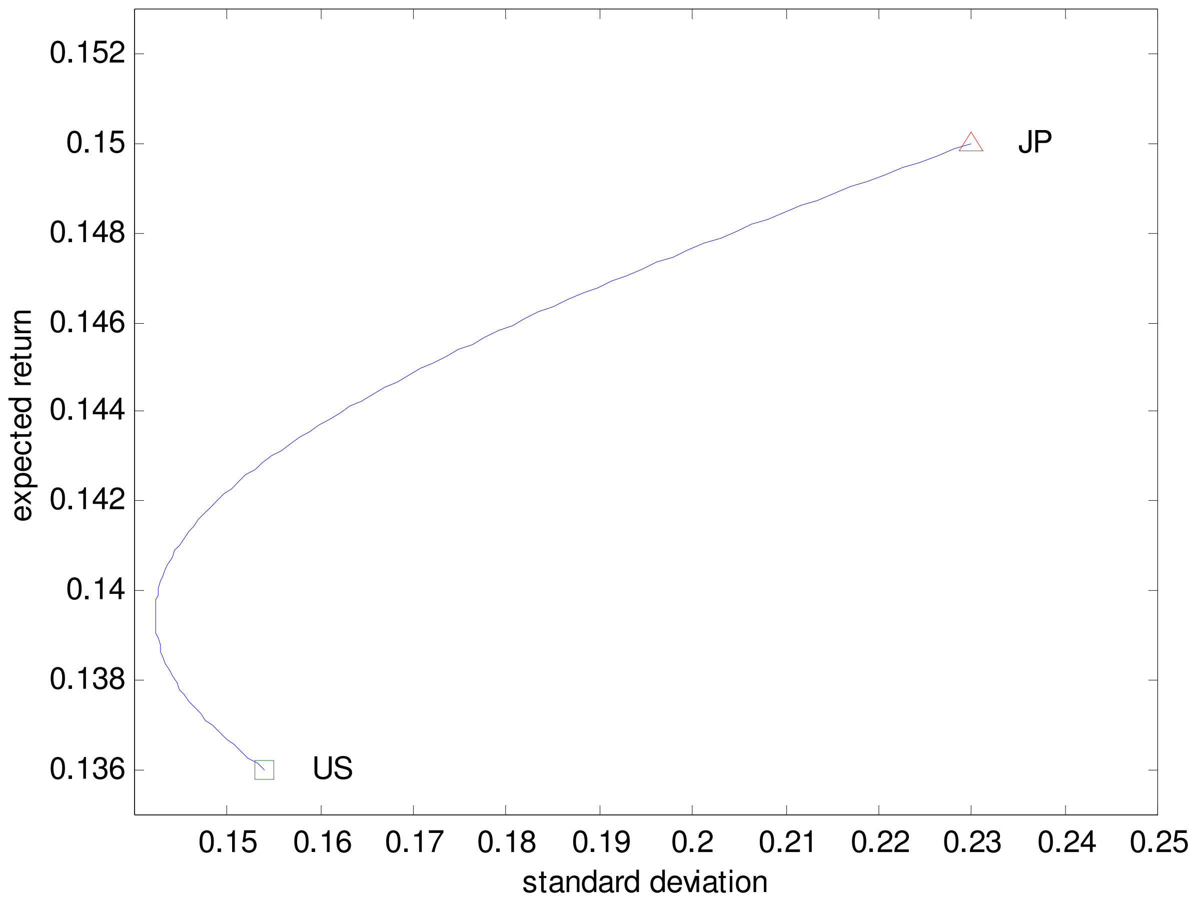
- Let ω be the weight in the US, and 1-ω the weight in JP.
- The expected return of the portfolio is:

  $E(r_p) = \omega*0.136 + (1-\omega)*0.150$
- The variance of the portfolio return is:

  $var(r_p) = \omega^2*(0.154)^2 + (1-\omega)^2*(0.230)^2$

  $+2*\omega*(1-\omega)*0.27*0.154*0.230$
- What happens when we vary ω?

Varying the Portfolio Weights gives: The Investment Opportunity Set

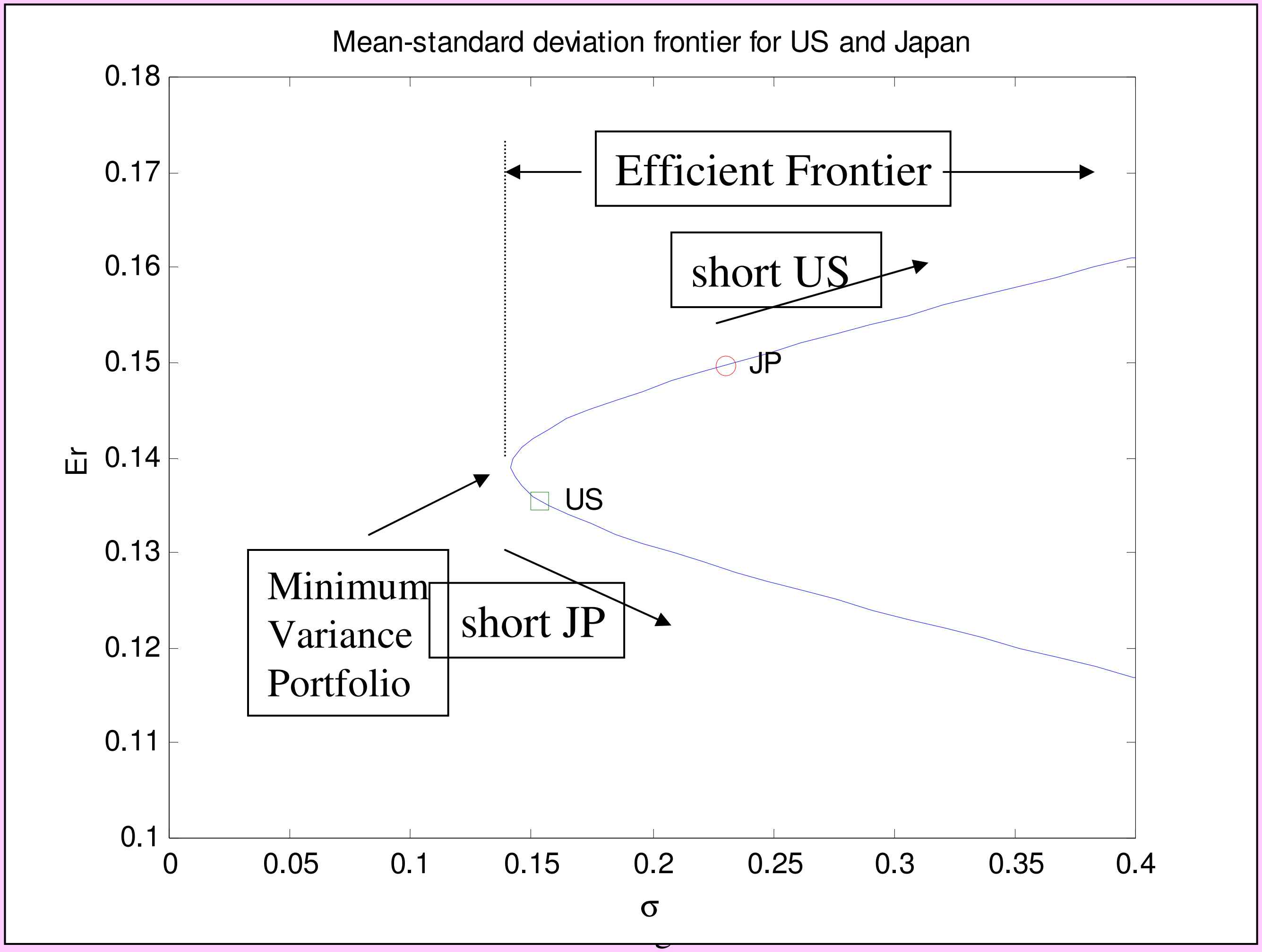


w	mean	volatility
0.0	0.150	0.230
0.1	0.149	0.212
0.2	0.147	0.195
0.3	0.146	0.179
0.4	0.144	0.166
0.5	0.143	0.155
0.6	0.142	0.147
0.7	0.140	0.143
0.8	0.139	0.143
0.9	0.137	0.146
1.0	0.136	0.154

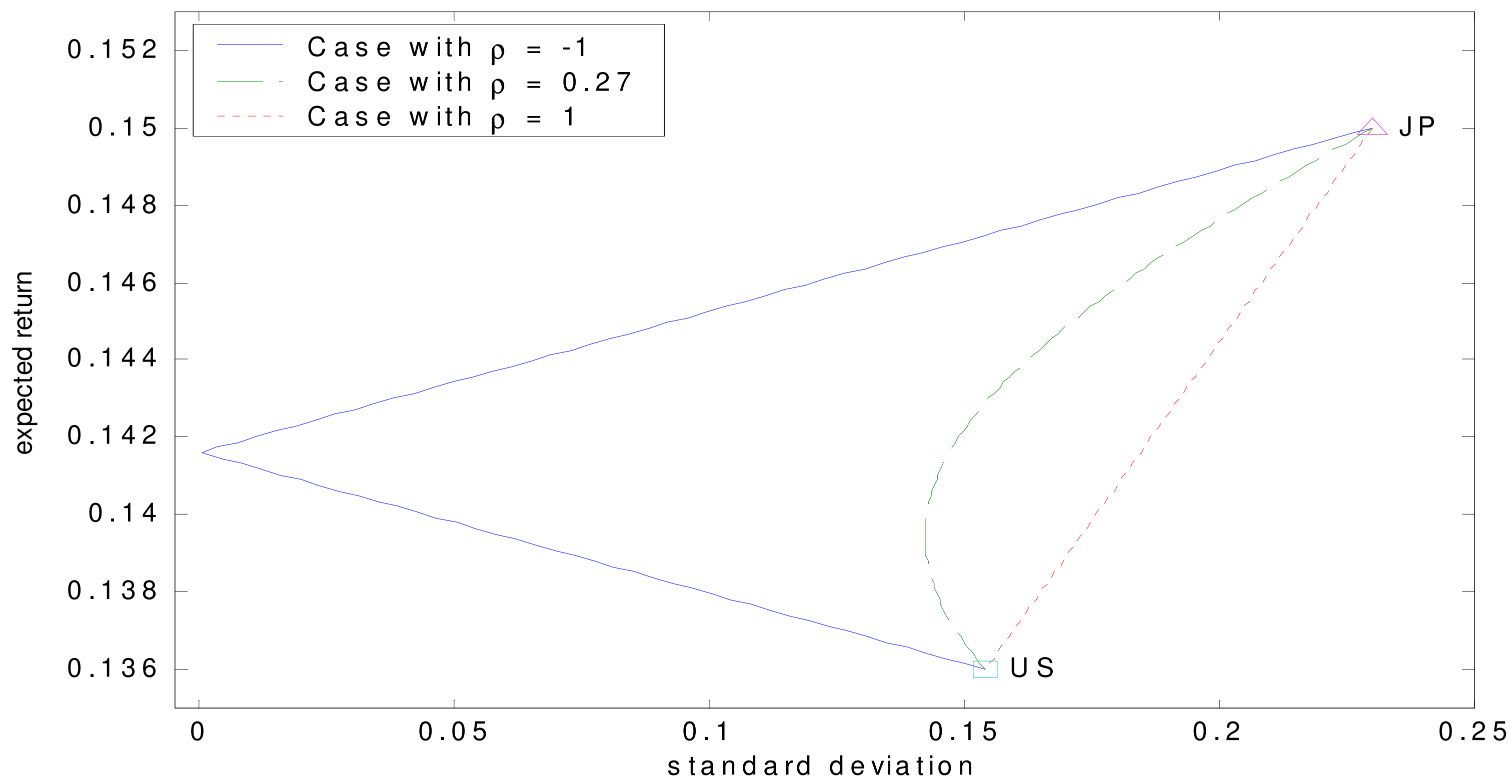
Portfolio Terminology

- The investment opportunity set consists of all available risk-return combinations.
- An efficient portfolio is a portfolio that has the highest possible expected return for a given standard deviation
- The efficient frontier is the set of efficient portfolios. It is the upper portion of the minimum variance frontier starting at the minimum variance portfolio.
- The minimum variance portfolio (mvp) is the portfolios that provides the lowest variance (standard deviation) among all possible portfolios of risky assets.

Portfolio Terminology



Investment Opportunity Set with Varying Correlations



Optimal Portfolio Choice with 2 Risky Assets

- Any (mean-variance) investor should choose an efficient portfolio to benefit from diversification.
- The specific choice depends on the investor's risk aversion
- A more risk-averse investor should choose a portfolio with
  - lower risk
  - lower expected return

C. Asset Allocation with 2 Risky and a Riskless Asset

Optimal Portfolios

- Putting together all our tools, we are now ready to construct optimal portfolios.

- This is a three-step process:
  1. Identify the best possible (most efficient) risk / return combinations available from the universe of risky assets
  2. Determine the optimal portfolio of risky assets that supports the steepest CAL
  3. Choose the optimal portfolio for the investor based on risk aversion by mixing the risk free asset with the optimal risky portfolio.

Efficient Diversification: STEP 1

- First, we want to determine the efficient frontier.

- *Efficient Frontier*: the set of portfolios of risky assets that maximize expected return $E(r_P)$ for each level of portfolio risk σ_P.

- We did it in the previous Set of Slides. (Compute the expected return and standard deviation of a typical portfolio, and put them in a diagram, for all possible weights)

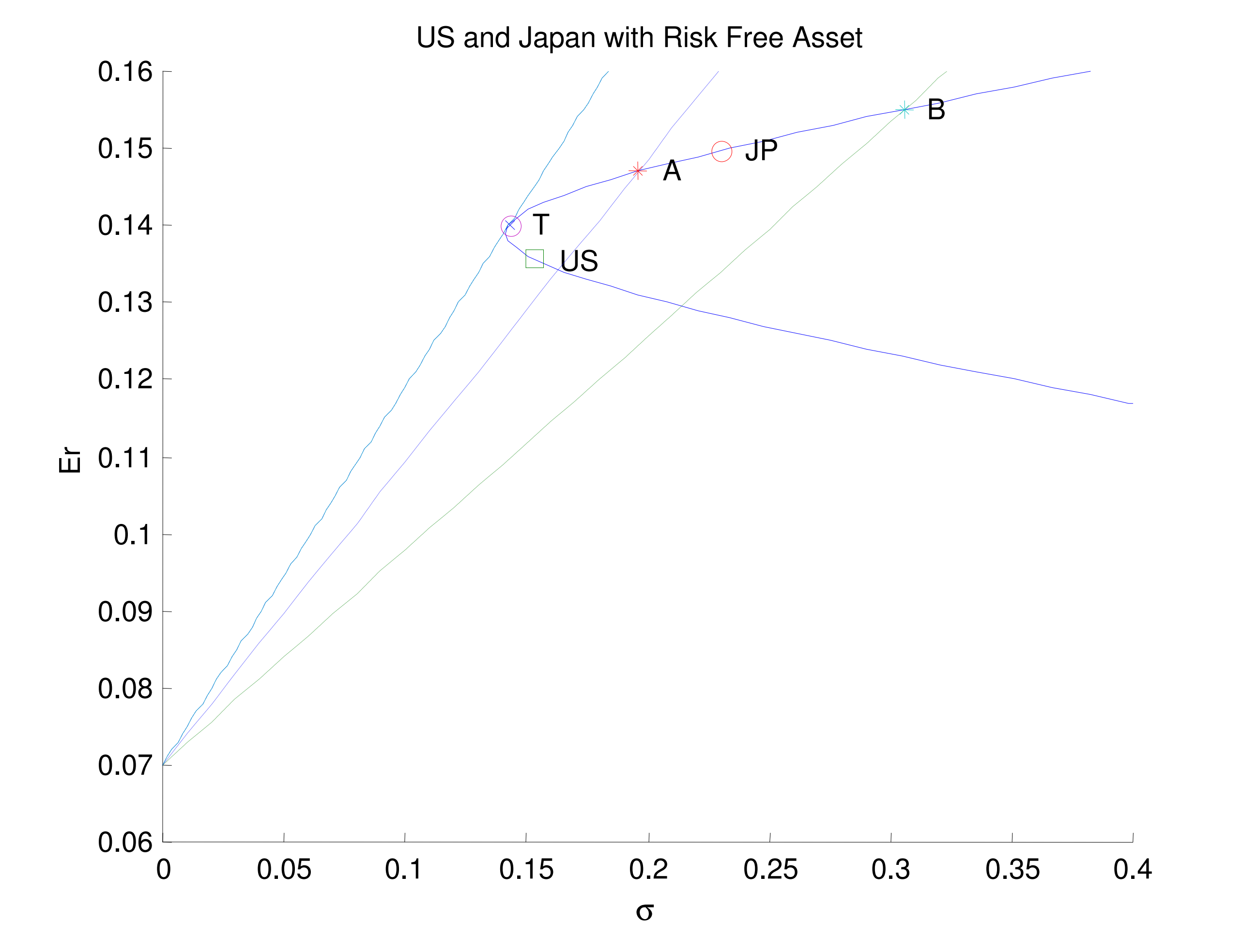
Efficient Diversification: STEP 2

- Once we bring in the risk free asset, we need to search for the CAL with the highest reward to variability ratio that connects to one of the risky portfolios.

$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

- This CAL will be tangent to one portfolio of risky assets on the efficient frontier. This portfolio is the optimal risky portfolio.

Optimal CAL



The tangency portfolio maximizes the Sharpe ratio:

$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

Efficient Diversification: STEP 3

- The investor chooses the appropriate mix between the optimal risky portfolio and the risk free asset based on his or her own risk aversion.

- But notice that the optimal risky portfolio is the same for all investors (Separation principle).

Two-Fund Separation

- All investors hold combinations of the same two "mutual funds":
  - The risk-free asset
  - The tangency portfolio

- An investor's risk aversion determines the fraction of wealth invested in the risk-free asset

- But, all investors should have the rest of their wealth invested in the tangency portfolio.

Asset Allocation – An Example

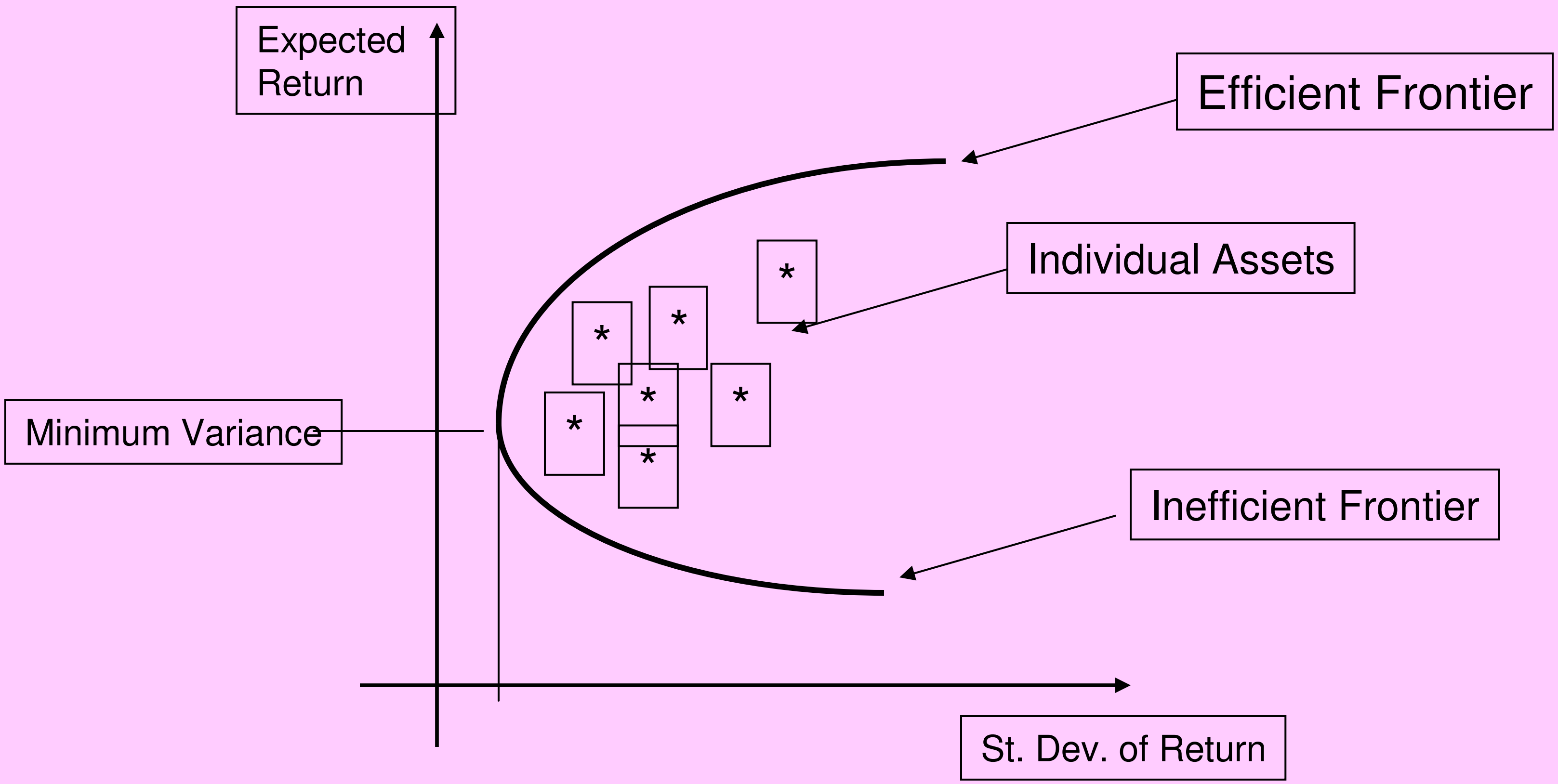
- We have $100,000 to invest in T-Bills, US and JP. We want an expected return of 12%. Assume that the tangency portfolio is consisted of 30% US and 70% JP.

- How much should we invest in the risky portfolio?
- How much should we invest in the risk free asset?
- What would be the risk of this portfolio?
- Would a combination of only US and JP that gives the same expected return have more or less risk than the portfolio composed of the risky portfolio and the risk free asset?

Example - Solution

D. Asset Allocation with Many Risky and a Riskless Asset

Investment Opportunity Set with Many Assets



Optimal Risky Portfolio: Reality Check

- The optimal risky portfolio may differ across individuals because of
  - Input data
  - Tax considerations
  - individual philosophies
  - Liquidity considerations

In practice, individuals' risky portfolios are not as different as one may think (mutual funds)